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Assessment of the behavior of weakly and highly nonlinear friction-driven oscillatory systems

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Abstract

The behavior of weakly and highly nonlinear oscillatory systems driven by friction is investigated in this work with the approximate and numerical solutions developed via a piecewise-constant approach. The solutions developed for the systems are continuous everywhere on the time range desired. It is found in the research that the properties of the weakly and highly nonlinear systems exhibit great differences, though the governing equations of the two systems employ identical system parameters. The approximate solutions developed can be conveniently implemented for the purpose of analytically and numerically studying the properties of the system with numerous system parameters and various initial conditions. The highly nonlinear system is found physically much involved and its behavior is much complex in comparing with that of the weakly nonlinear system. Based on the approximate solutions developed for the highly nonlinear system, recurrence relations are generated for numerical calculations.

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1. Introduction

Friction forces are the sources of oscillations in many types of machinery. Nonlinearities and uncertainties generated by friction forces are commonly presented in modeling and analyzing the

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motions of the mechanical systems involving components contacts and interfaces. Froude pendulum is a typical example of the oscillatory systems driven by the frictions due to the interactions of components of the pendulum. This type of pendulum is nonlinear and rich of complex behavior of nonlinear dynamics. In recent years, the quantitative estimates of the dynamics of the friction process in Froude pendulum were reported by Maryuta [1] and the stability of a free and weakly nonlinear Froude pendulum was examined by Butenin and Kunitsyn [2,3]. Thomas and Ambika [4] investigated the driven pendulum by the harmonic balancing method. The stability analysis was carried out by transforming the system of equations to the linear Mathieu equation. Dai and Singh [5] recently studied a Froude pendulum driven by a sinusoidal exertion. The periodic, quasiperiodic and chaotic behavior of the pendulum was analyzed through a numerical approach. Oscillation of a self-excited Froude pendulum subjected to external forcing were analyzed by Litak et al. [6] with utilization of the multiple time scale method.

Froude pendulum is usually used in the mechanical literature as an example illustrating the generation of self-oscillations in mechanical systems with friction; thought the generation and maintenance of the self-exited oscillation have not been verified through a physical experiment. According to the current literature, the theoretical studies on the oscillations of a Froude pendulum are mainly concentrated on the approximate analytical solutions to the weakly nonlinear systems of Froude pendulum. A systematic study on the motion of a highly nonlinear Froude pendulum is still in a lack.

In this paper, the authors attempt to analyze the motion of a Froude pendulum and provide the approximate and numerical solutions to nonlinear dynamical system with a piecewise-constant and P-T method. The properties of motion of a weakly nonlinear system are first analytically and numerically investigated. Comparison of the analytical and numerical results with the analytical solution given by Van der Pol's method is also performed. The behavior of oscillation of highly nonlinear Froude pendulum systems is then numerically examined and compared with those obtained for the weakly nonlinear systems with various initial conditions and numerous system parameters.

2. Equation of motion for a free Froude pendulum

The Froude pendulum is a pendulum suspended from a horizontal rotating shaft of circular cross-section by a bearing pivot. The equation of motion for a Froude pendulum can be expressed in the following form:

$$I\ddot{\theta} + c\dot{\theta} + mgl\sin\theta = M(\Omega - \dot{\theta}) \tag{1}$$

where the frictional torque of a Froude pendulum is believed to have a relation to the slipping angular velocity $\dot{\theta}$ and is expressible in a functional form $M(\Omega - \dot{\theta})$ [1–3]. In the above equation, m is the mass of the pendulum, I is the total moment of inertia of all rotating components of the pendulum, c is the viscous coefficient due to the resistance of the air, l is the distance from the axis of rotation to the center of gravity of the pendulum, Ω is the angular velocity of the rotating shaft. The function $M(\Omega - \dot{\theta})$ in the equation can be expanded in a power series as follows:

$$M(\Omega - \dot{\theta}) = M(\Omega) - M'(\Omega)\dot{\theta} + \frac{1}{2}M''(\Omega)\dot{\theta}^2 - \frac{1}{6}M'''(\Omega)\dot{\theta}^3 + \cdots \tag{2}$$