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A semi-discretization method for delayed stochastic systems

O. Elbeyli [a], J.Q. Sun [a,*], G. Ünal [b]

[a] Department of Mechanical Engineering, University of Delaware, 229 Spenceer Lab, Newark, DE 19716, USA
[b] Faculty of Sciences, Istanbul Technical University, Maslak 80626, Istanbul, Turkey

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Abstract

Time delayed linear stochastic systems with state feedback is investigated. An extension of the semi-discretization method to stochastic systems is developed. Mappings of the second order moments are formulated by using the exact drift mapping and direct Itô integration. It is found that the exact drift mapping yields better results compared to the direct Itô integration. Stability boundaries and steady state second order moments are studied with the proposed method. The results are compared with the known exact solutions or the simulations. Good agreement among the results is found, which strongly validates the proposed method.

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1. Introduction

Time delay comes from different sources and often leads to instability or poor performance in control systems. Other than a few exceptional cases, delay is undesired and control strategies to eliminate or minimize its unwanted effects have to be employed. Effects of time delay on the stability and performance of deterministic control systems have been a subject of many studies. For example, Yang and Wu [1] and Stepan [2] have studied structural systems with time delay. A study on stability and performance of feedback controls with multiple time delays is reported in

* Corresponding author. Tel.: +1-302-831-8686; fax: +1-302-831-3619.
E-mail address: sun@me.udel.edu (J.Q. Sun).

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[3] by considering the roots of the closed loop characteristic equation. For deterministic delayed linear systems a survey of recent methods for stability analysis is presented in [4].

There is a growing interest in the stochastic systems with time delay. An effective Monte Carlo simulation scheme that converges in a weak sense is presented by Kuchler and Platen [5]. Buckwar [6] studied numerical solutions of Itô type differential equations and their convergence where the system considered has time delay both in diffusion and drift terms. Guillouzic et al. [7] studied first order delayed Itô differential equations using a small delay approximation and obtained PDFs as well as the second order statistics analytically. Frank and Beek [8] obtained the PDFs using FPK equation for linear delayed stochastic systems and studied the stability of fixed point solutions in biological systems. State feedback stabilization of non-linear time delayed stochastic systems are investigated by Fu et al. [9] where a Lyapunov approach is used.

The delayed systems are studied using discretization techniques with an extended state vector. For example, Pinto and Goncalves [10] fully discretized a non-linear SDOF system to study control problems with time delay. Klein and Ramirez [11] studied MDOF delayed optimal regulator controllers with a hybrid discretization technique where the state equation was partitioned into discrete and continuous portions. Another powerful discretization method is the semi-discretization method. It is a well established method in the literature and used widely in structural and fluid mechanics applications. Recently, the method is applied to delayed deterministic systems by Insperger and Stepan [12]. They studied high dimensional multiple time delayed systems in [13]. The method can be extended to control systems with delayed feedback. We have studied the effect of various higher order approximations in semi-discretization on the computational efficiency and accuracy. The merit of the semi-discretization method as introduced by Insperger and Stepan [12] lies in that it makes use of the exact solution of linear systems over a short time interval to construct the mapping of a finite dimensional state vector for the system with time delay.

In this paper we apply the semi-discretization method to the systems with time delay and subject to additive and multiplicative stochastic disturbances. The paper is organized as follows. In Section 2, we introduce the mathematical background and the semi-discretization method for stochastic systems with time delay. In Section 3, we demonstrate the method with an example, and compare the results with known analytical solutions. Responses of the system subject to additive and multiplicative stochastic disturbances are also presented. Section 4 concludes the paper.

2. Mathematical background

2.1. Semi-discretization

Consider a stochastic system in the Stratonovich sense,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{x}(t-\tau), t) + \mathbf{G}(\mathbf{x}, t)\mathbf{W}(t), \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^r$, $\mathbf{W} \in \mathbf{R}^p$, $\mathbf{f}$ describes the system dynamics with time delay, and $\mathbf{G} = \{G_{ij}\}$ is a matrix determining the parametric and external random excitations. $W_i(t)$ are delta correlated Gaussian white noise processes with $E[W_i(t_1)W_j(t_2)] = 2\pi K_{ij}\,\delta(t_1 - t_2)$. Here, we extend the semi-discretization method as presented in [13] to stochastic systems.