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Engineering Failure Analysis 12 (2005) 362–375
ENGINEERING FAILURE ANALYSIS
www.elsevier.com/locate/engfailanal

Stress intensity factors for cracks emanating from a triangular or square hole in an infinite plate by boundary elements

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Received 19 July 2004; accepted 4 September 2004
Available online 5 November 2004

Abstract

This paper concerns stress intensity factors of cracks emanating from a triangular or square hole in an infinite plate subjected to internal pressure calculated by means of a boundary element method, which consists of constant displacement discontinuity element presented by Crouch and Starfied and crack tip displacement discontinuity elements proposed by the author. In the boundary element implementation the left or the right crack tip displacement discontinuity element is placed locally at corresponding left or right crack tip on top of the constant displacement discontinuity elements that cover the entire crack surface and the other boundaries. Numerical examples are included to show that the method is very efficient and accurate for calculating stress intensity factors of plane elasticity crack problems. Specifically, the numerical results of stress intensity factors of cracks emanating from a triangular or square hole in an infinite plate subjected to internal pressure are given.

Keywords: Stress intensity factor; Boundary element method; Fracture mechanics; Cracks; Holes

1. Introduction

The boundary element method (BEM) has proven to be a powerful numerical technique, which has certain advantages over the domain-based method such as the finite element method. The most important feature of the BEM is, as is well known, that it only requires discretization of the boundary rather than the domain.

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1350-6307/$ - see front matter
doi:10.1016/j.engfailanal.2004.09.008

Various formulations of boundary integral equation methods have been developed for elastic fracture mechanics problems. These formulations differ from each other mainly because of the different approaches used in dealing with the singularity of stress near a crack tip and the geometry identity of the surfaces of a crack. The standard boundary element formulation, when regarding the cracks as narrow slits with upper and lower surfaces slightly separated, degenerates for flat cracks and is simply not appropriate for numerical modeling [1]. This degeneracy is linked to the ill-posed nature of problems with two coplanar surfaces. Several different formulations have been proposed to avoid this fundamental limitation. The first one is the Green's function method [2], which has the advantage of avoiding crack surface modeling and gives excellent accuracy. It is however restricted to fracture problems involving very simple crack geometries for which analytical Green's functions can be obtained. The second one is the multi-domain technique [3]. The advantage of this approach is its ability to model cracks with any geometric shape. The disadvantage is an artificial subdivision of the original domain into several subdomains, thus resulting in a large system of equations. The third approach is the displacement discontinuity method [4,5]. Instead of using the Green's displacements and stresses from point forces, the displacement discontinuity method uses Greens functions corresponding to point dislocations (i.e., displacement discontinuity). The fourth approach is the so-called dual boundary element method [6,7], where the displacement integral equation is collocated on the no-crack boundary and on one side of the crack surface while the traction integral equation is collocated on the other side of the crack surface. For each formulation, in order to model the singularity of stress near a crack tip, options are available such as building in the crack tip stress singularity [8], using the quarter-point boundary element [3], and strategically refining the near-crack-tip nonsingular element. Further details on elastic crack analysis by the boundary element method are given in [9–11].

In a recent paper [12], special crack tip displacement discontinuity elements are developed by the author to model the singularity of stresses near a crack tip and stress intensity factors are calculated numerically by using the displacement discontinuity components at the midpoints of these crack-tip elements. In this paper, the crack tip elements together with constant displacement discontinuity element presented by Crouch and Starfied [5] is made to present a boundary element method to calculate numerically stress intensity factors for complex crack problems in a general plane elasticity body. In the boundary element implementation the left or the right crack tip element is placed locally at corresponding left or right crack tip on top of the constant displacement discontinuity elements that cover the entire crack surface and the other boundaries. The method is called a hybrid displacement discontinuity method (HDDM). Numerical examples are included to show that the method is very efficient and accurate for calculating stress intensity factors of plane elasticity crack problems. For cracks emanating from a triangular or square hole in an infinite plate under tension, Murakami [14] used the body force method to calculate their stress intensity factors, see documented literature [15]. To the author's knowledge, stress intensity factors of cracks emanating from a triangular or square hole in an infinite plate subjected to internal pressure was not obtained. Newman [16] used the boundary collocation method to calculate stress intensity factors of cracks emanating from a circular hole in an infinite plate subjected to internal pressure. In this paper, the author concerns with stress intensity factors of cracks emanating from a triangular or square hole in an infinite plate subjected to internal pressure by using the HDDM. A large number of numerical results are given.

2. Formulations and numerical procedure

A boundary element method used here to analyze the complex plane crack problems commonly occurring in engineering consists of constant displacement discontinuity element presented by Crouch and Starfied [5] and crack tip displacement discontinuity elements by the author [12].