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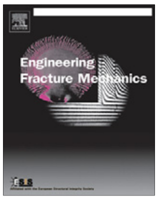

A statistical approach for transferring fracture events across different sample shapes

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ABSTRACT

The paper investigates the capability of a novel calibration method to predict accurately fracture events across different sample shapes at low temperatures. It is shown that the emergence of a threshold Weibull stress in the Weibull stress distribution is inherent in the fundamental assumptions of the Beremin model. The mathematical concept underlying the suggested calibration method is the correlation between the probability distributions of the fracture loads and the associated Weibull stresses. The calibration procedure is demonstrated using fracture data obtained in tests conducted at a test temperature of −150 °C on specimens fabricated of A533B ferritic steel. In contrast to the values found in the literature, the calibrated Weibull modulus is small and ranges from 2 to 4. The proposed methodology is straightforward to apply and yields reliable predictions of the failure probabilities of samples of different shapes.

1. Introduction

Catastrophic cleavage fracture is one of the most severe failure modes in steel structures [1]. There exist several approaches to assess the integrity of mechanical structures constructed of ferritic steels. The global approach results directly from elastic–plastic fracture mechanics and describes the fracture resistance in terms of the crack tip parameters K and J [2].

The Beremin model [2] is a statistical model for cleavage fracture based on the micromechanisms of failure at a local scale and incorporates a weakest link concept. This local approach was developed, in 1983 by a French research group, to explain the large scatter observed in the cleavage resistance in the lower shelf region. It has since received a great deal of attention from researchers and industry alike. The original model and its extensions are the most widely applied approaches to predict cleavage fracture [3]. The success of the Beremin model stems from its simplicity of application and ability to predict the scatter in cleavage stress measurements, although recent arguments suggest that its underlying theory of microcrack initiation and propagation is too simple [4–7].

The Beremin model introduces a statistical variable that follows a Weibull distribution and is called the Weibull stress. The hypothesis is that there exists a unique Weibull stress distribution that enables prediction of the failure probability for any given fracture geometry. The values of the Weibull stress at different load levels have to be calculated from a Finite Element Analysis (FEA). Inconveniently, in the definition of the Weibull stress, a parameter is incorporated that has to be determined based on measured fracture data. This parameter is also the shape parameter of the probability distribution of the Weibull stress. Reported values for this parameter range from 10 to 50 [8,9] which is not in agreement with the fundamental assumptions of the Beremin model and the findings reported by Kroon and Faleskog [1].

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Nomenclature

$(\cdot)$	to be replaced by a variable or expression
$\widehat{(\cdot)}$	indicates parameters estimated from experimental data
$\widetilde{(\cdot)}$	indicates predicted parameters
$(\cdot)_{\text{th}}$	indicates location parameter in a Weibull distribution
$(\cdot)_0$	indicates scale parameter in a Weibull distribution
$m_{(\cdot)}$	indicates shape parameter in a Weibull distribution
a, b	slope and intercept of straight line given by $g^{-1}(\cdot)$
$F(\cdot)$	cumulative distribution function
$g(\cdot)$	linear function describing the relationship between σ_w and L
J	contour integral
K	stress intensity factor
K_{min}	location parameter in the Weibull distribution of K
L	fracture load
m_σ	Weibull modulus
ℓ_{min}, ℓ_{max}	smallest and largest microcrack that will occur in the material
N	sample size
$\mathbb{P}(\cdot)$	probability of expression $(\cdot)$
p-value	probability value
R^2, R^2_{max}	squared and maximum squared regression coefficient
V_0	reference volume
X, Y	random variables
α	exponent in the size distribution of microcracks
$\Delta(\cdot)$	uncertainty in parameter $(\cdot)$
σ_c	critical stress (maximum principal stress)
$\sigma_{c,\text{min}}$	minimum critical stress corresponding to largest microcrack
σ_w	Weibull stress
$\sigma_{w,\text{min}}$	minimum value of Weibull stress σ_w
σ_w^*	modified Weibull stress
m_L, L_0, L_{th}	fracture load parameters
m_σ, σ_0, $\sigma_{w,\text{th}}$	Weibull stress parameters
m_σ, σ_0^*, $\sigma_{w,\text{th}}$	modified Weibull stress parameters
Ω, $\overline{\Omega}$	volume and average volume of the fracture process zone
FEA	Finite Element Analysis

It has become common to employ a three-parameter Weibull distribution instead of the two-parameter form derived in the original paper [2] because it improves the fit to the experimental data [8,10,11]. Despite its wide application, the use of the three-parameter Weibull distribution has not been justified theoretically [8] and is considered as an engineering distribution [10]. Occasionally, a modified expression of Weibull stress is applied, by directly incorporating a third parameter into the definition of the Weibull stress. The third parameter is a threshold value, below which fracture cannot occur, and naturally results from the Beremin model by proposing that the distribution of the size of microcracks in a material has an upper and lower limit on microcrack size [1,5,12,13].

The problem of estimating three model parameters simultaneously is difficult and so far a rational calibration algorithm has not been achieved [14]. One calibration procedure that enables transferability of toughness values for specimens of different types, sizes and loading conditions [7,8] requires fracture toughness data from two sets of specimens that exhibit different constraint levels at fracture. This eliminates the ambiguity in the parameter estimates from the standard iterative procedure [15].

The paper, and the local approach in particular, are based on a fundamental idea often encountered in mathematical physics. If an experiment is repeated again and again under the same macroscopic conditions, it is impossible to control the microscopic details as well, such that it may be expected to observe a range of different outcomes. Performing the same fracture test on identical fracture specimens will result in a range of measured fracture loads or fracture toughness values. Realistically, the specimens will differ somewhat from each other due to the non-ideal manufacturing process. However, this will be neglected here. Another crucial point in this paper is the assumption that the value of the fracture toughness is linearly dependent on the applied fracture load. Using the fracture load rather than the fracture toughness in the proposed calibration procedure provides an unambiguous choice of loading parameter and is a simplification that is employed in order that notched specimens, where the fracture toughness is not rigorously defined [16], can be considered as well. The linear dependence can be deduced from the common equations relating the fracture toughness and the fracture load [17].

In this paper, it will be shown how the three-parameter Weibull distribution emerges from the original Beremin models without the requirement for additional assumptions. A rational parameter calibration algorithm is proposed, utilising only