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Fracture instability and limit strength condition in structures with re-entrant corners

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Abstract

When considering a structural element with a re-entrant corner, the experimental analysis shows how the fracture strength increases with the angle of the corner. Thus, the strength increases with a decrease of the mass of the structure, in contrast to what we are used to observe in different kind of collapses, e.g., plasticity. To predict this behaviour, a non-local theory, basically based on the Novozhilov's hypothesis of existence of a fracture quantum, is herein presented. Theoretical predictions for the strength of finite structures (e.g., finite plates under tension or beams under bending) by varying both angle and relative depth of the corner are presented: accordingly, simple formulas, useful in the design of such structures, are provided. The theory is then compared with experimental and numerical results, showing a relevant agreement.

Keywords: Fracture; Strength; Re-entrant corners; Non-local fracture; Fracture quantum

1. Introduction

Non-local fracture criteria are powerful methods in the study of crack propagation. In [1] Novozhilov presents a non-local tensional criterion based on the existence of a fracture quantum. He identified the fracture quantum with the atomic size of the crystalline lattice. On the other, his criterion is basically a non-local stress criterion, and can treat also materials in which the link to the atomic structures is absent, as shown by the Novozhilov's apprentices and recently emphasized in [2]. We apply this method to study analytically the problem of the strength against fracture of structures containing re-entrant corners.

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Nomenclature

σ_{ij}	stress-field
σ	far-field stress
σ_f	stress of failure
σ_u	strength of the material
K_I	stress-intensity factor for the Mode I
K_{IC}	critical stress-intensity factor for the Mode I
r and φ	the polar co-ordinates
α	power of the stress singularity
S_{ij}	function describing the angular profile of the stress-field
f	shape function
g	generalized shape function for $\gamma = \pi$
b	structure width
a	defect length
γ	re-entrant corner angle
l	structure length (of the three-point bending)
t	structure thickness
P	applied load (on the three-point bending specimen)
P_{CR}	critical load
s	brittleness number
d_0	fracture quantum
Superscript *	refers to generalized quantities for re-entrant-corners
Superscript π	refers to generalized quantities evaluated for $\gamma = \pi$

Since the pioneer paper [3] the problem of stress intensification at the vertex of re-entrant corners has not been sufficiently addressed if compared with its considerable practical importance. Shapes and sizes of notches or re-entrant corners in structural components are studied more frequently than shapes and sizes of cracks. In spite of this, fracture mechanics applied to (long) sharp cracks [4,5] has been broadly developed in the last three decades, even if only as a special case of the more general problem of re-entrant corners.

The investigation on stress intensification at the vertex of re-entrant corners carried out at CSIRO Australian Forest Production Laboratory, Division of Building Research is very notable. In [6] the size scale effects in structures with re-entrant corners due to the presence of a stress-singularity were investigated and noted that they occur only when the member sizes are sufficiently large. Consequently, such scale effects may not appear in scaled-down laboratory testing. The work presented in [6] was continued in [7] extending conventional finite element procedures to non-zero angle notch problems. The author considered also the problem of crack initiation at corners of openings in walls and examined the effect of beam size on the sharp crack propagation in concrete [8].

In [9] the determination of realistic measures for the peak local stresses occurring at sharp re-entrant corners in plates under remote transverse loading has been considered. The authors took up the singular character of re-entrant corners and carried out experimental investigation on classical stress concentration. Then, the Reciprocal Work Contour Integral Method was used to obtain the stress singularity at the tip of corner configurations [10]. In this way, the numerical analysis of a lap joint with $\pi/2$ corner angles in mixed mode loading has been performed.