Precision Engineering 38 (2014) 525–537

Contents lists available at ScienceDirect

Precision Engineering

journal homepage: www.elsevier.com/locate/precision

Modeling and generating parallel flexure elements

Jonathan B. Hopkins*, John J. Vericella, Christopher D. Harvey

Lawrence Livermore National Laboratory, 7000 East Avenue, Livermore, CA 94551, USA

ARTICLE INFO

Article history:
Received 31 January 2013
Received in revised form 31 January 2014
Accepted 4 February 2014
Available online 12 February 2014

Keywords:
Flexure elements
Compliant members
Flexible joints
Elements of constraint
Flexure systems
Screw theory
Freedom and Constraint Topologies
FACT

ABSTRACT

This work introduces the principles necessary to model and generate parallel flexure elements (i.e., compliant members or flexible joints) that may be used to synthesize next-generation precision flexure systems. These principles are extensions of the Freedom and Constraint Topologies (FACT) synthesis approach, which utilizes geometric shapes to help designers synthesize flexure systems that achieve desired degrees of freedom (DOFs). Prior to this paper, FACT was limited to the design of flexure systems that consisted primarily of simple wire or blade flexure elements only. In this paper, the principles are introduced that enable designers to use the same shapes of FACT to synthesize parallel flexure elements of any geometry, including new and often irregularly-shaped elements (e.g., hyperboloids or hyperbolic paraboloids). The ability to recognize such elements within the shapes of FACT, therefore, enables designers to consider a larger body of solution options that satisfy a broader range of kinematic, elastomechanical, and dynamic design requirements. Example flexure systems that consist of flexure elements, generated using this theory, are provided as case studies.

1. Introduction

Flexure systems consist of rigid bodies that are joined together by flexure elements [1] (i.e., compliant members or flexible, spring-like joints). These elements are directionally compliant and thus behave like bearing elements in that they guide the system's rigid bodies to move in prescribed directions called degrees of freedom (DOFs) via elastic deformation. As such, flexure systems (i) achieve high resolution and repeatability, (ii) require no lubrication, (iii) experience minimal friction and wear, (iv) are easily maintained, and (v) often cost significantly less than other competing precision bearing technologies (e.g., magnetic or air bearings).

The rigid bodies within most flexure systems are joined together by simple wire, blade, and/or living hinge flexure elements [2–5] like those shown in Fig. 1A. The wide-spread use of these common flexure elements is due to the fact that they (i) possess DOFs that are easy to visualize, (ii) are relatively easy to fabricate, and (iii) are often the only flexure element options to which designers have been exposed. The demand, however, for precision flexure systems that possess greater kinematic, dynamic, and elastomechanic versatility is growing as flexure-based applications are becoming more sophisticated.

This paper provides the theory necessary to model and generate a significantly larger variety of flexure elements with geometries that are often unconventional, like those shown in Fig. 1B, that satisfy the requirements of such sophisticated applications. These applications include multi-axis spatial micro/nano-manipulation and assembly stages, advanced flexure bearing systems that guide rigid bodies along complex motion paths, and microstructural architectures that achieve unusual and often superior properties compared with most natural materials (e.g., negative thermal expansion coefficient and Poisson's ratio) [6–9]. Such architectures are generally fabricated using additive micro/nano-3D printing technologies such as micro-projection stereolithography [10–12] because their flexure element constituents are small and often irregularly-shaped and are thus not suited for conventional fabrication processes (e.g., waterjet, wire EDM, and milling). Although this paper enables the generation of irregularly-shaped elements like those in Fig. 1B, it also enables the generation of new elements with regular features that possess unique constraint characteristics and may be fabricated using conventional processes. As progress toward high-resolution multi-material additive fabrication technology advances, however, flexure system designs for these and other applications will be driven more by performance requirements and less by fabrication limitations.

The theory used to model and generate the flexure elements of this paper is an extension of the Freedom and Constraint Topologies (FACT) synthesis approach [13–15]. FACT utilizes a comprehensive library of geometric shapes that represents the mathematics

* Corresponding author. Tel.: +1 617 470 4298.
E-mail address: jonathanbhopkins@gmail.com (J.B. Hopkins).

http://dx.doi.org/10.1016/j.precisioneng.2014.02.001
0141-6359/

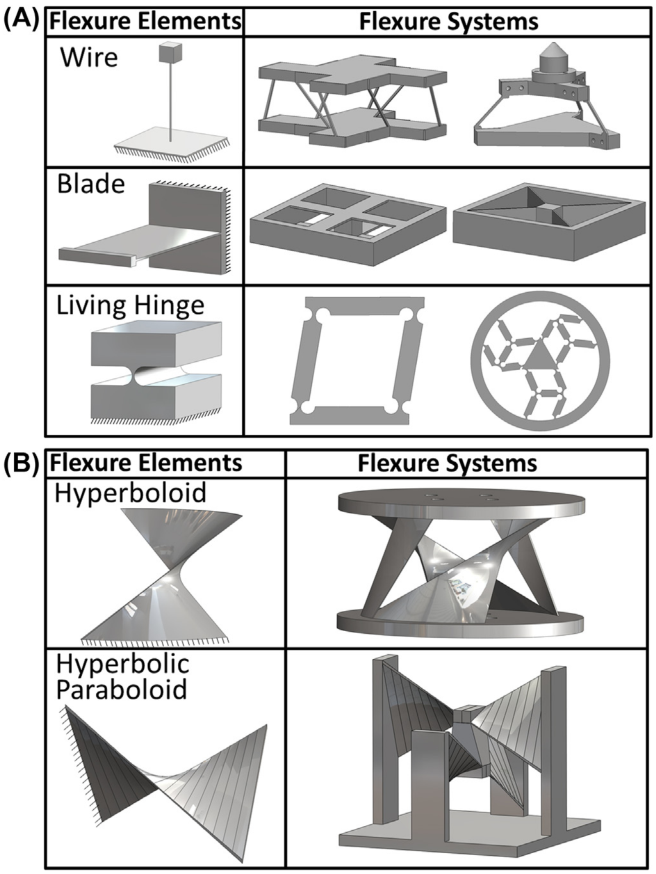

Fig. 1. Common parallel flexure elements and example systems that consist of these elements (A), and irregularly shaped parallel flexure elements with example systems (B).

of screw theory [16–21] to help designers synthesize flexure systems. One set of shapes represents the flexure system's desired DOFs while another set of shapes represents the regions of space from which flexure elements may be selected that enable the system to achieve those desired DOFs. In this way, designers may rapidly consider and compare a multiplicity of design solutions before selecting the final design. The shapes of FACT thus enable designers to utilize the mathematics of screw theory in an intuitive way without requiring a mastery of its principles. Prior to the theory of this paper, however, FACT and other screw-theory-based synthesis approaches [22] were only capable of guiding designers in synthesizing flexure systems that consisted primarily of simple wire or blade flexures. This paper provides designers with the theory necessary to utilize the same shapes of FACT to help designers synthesize flexure systems constrained by a host of new and often irregularly-shaped parallel flexure elements (Fig. 1B).

Such elements provide designers with a larger body of solutions for achieving a given set of functional requirements. Suppose, for instance, a designer wished to synthesize a flexure system that possessed a single screw DOF (i.e., a translation along an axis coupled with a simultaneous rotation about the same axis). If the designer is restricted to only use wire, blade, or living hinge flexure elements to synthesize such a system, the solution space is greatly limited. If the designer is further restricted to only arrange these elements in a parallel configuration such that they directly connect a single rigid body to a fixed ground, it is not possible to achieve a single screw DOF using blade or living hinge flexures. With these restrictions, therefore, such a system may only be achieved using wire flexures. An example is shown on the left side of the first row of Fig. 1A under "Flexure Systems." This system consists of wire flexures and would achieve the desired screw DOF about an axis that is perpendicular to and passes through the center of the flat surface of the system's stage. If the designer is no longer restricted to only use wire, blade, or living hinge flexure elements, but could also use the flexure elements generated using the theory of this paper to achieve a similar screw DOF, the designer would have access to a larger body of parallel design solutions. These solutions would possess a broader variety of kinematic, elastomechanical, and dynamic characteristics. Two examples of such systems are found in Fig. 1B under "Flexure Systems."

The main contributions of this paper include: (1) Principles are introduced that enable designers to rapidly generate the geometry of any general parallel flexure element using the shapes of FACT. (2) Principles are also introduced that enable designers to rapidly identify the DOFs of any parallel flexure element in an intuitive and visual way. (3) The concept of 'order of constraint' is introduced as a way of helping designers control exact- or over-constraint in systems that consist of general parallel flexure elements. (4) The theory is introduced that enables designers to identify how well a parallel flexure element constrains its unwanted motions while permitting its intended DOFs based solely on how well constraint lines fit inside its geometry. (5) Twenty five unconventional parallel flexure elements are provided as an alternative to the classic wire, blade, and living hinge elements commonly used in most existing flexure systems. (6) The original steps of the FACT approach for synthesizing parallel flexure systems is updated such that these systems may be synthesized to include any parallel flexure element—not just wire and blade flexures. (7) Parallel flexure system case studies are designed and fabricated to demonstrate the utility of this paper's theory.

It is important to recognize, that although most flexure systems do consist of the common variety of wire, blade, and/or living hinge flexures, designers have implemented other differently-shaped flexure elements prior to the theory of this paper. Hale, for instance, designed a twisted flexure element that also possesses a screw DOF [23]. Serpentine flexure elements (i.e., curved wire flexures) are also becoming increasingly popular for use in various MEMS applications [24]. Curved blade flexure elements have also become popular for applications that require axisymmetric cylindrical packaging [25]. An exhaustive collection of existing flexure elements is provided in Howell [26]. Many of these existing flexure elements may also be modeled and analyzed using the theory of this paper.

2. Background principles

This section reviews the principles of FACT that are necessary to model and generate flexure elements.

2.1. Degrees of freedom

To better understand flexure elements, it is necessary to first understand how to model the DOFs that they permit. According to screw theory, there are three types of DOFs—translations, rotations, and screws. Each of these motions may be modeled using a 6×1 twist vector, **T** [16,17]. In this paper, translations are depicted as black arrows, rotations are depicted as red lines, and screws are depicted as green lines along and about which a rigid body may simultaneously translate and rotate according to its coupled pitch value, p.

2.2. Basic constraint element

The most basic flexure constraint element is a wire flexure. A wire flexure is a straight, long, slender beam that joins two rigid bodies together as shown in Figs. 2A and B. The function of a wire