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Compliance-based matrix method for modeling the quasi-static response of planar serial flexure-hinge mechanisms

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ABSTRACT

A matrix method is proposed to model the direct and inverse quasi-static response of constrained/over-constrained planar serial mechanisms with flexure hinges under bending, axial, and shear planar (three-dimensional) loading and small-deformations. The method uses a basic three-point compliance matrix corresponding to one rigid link and one adjacent flexure hinge that are subjected to one point load. This matrix connects the displacements at a point on the rigid link with the load that is applied at another point on it, and the deformations of the flexure hinge at its distal point. The quasi-static model of planar serial flexure-based mechanisms with multiple links under single/multiple point loading results from linearly superimposing all relevant hinge-link-load triads defined by their three-point matrices. A displacement-amplification planar device with right circularly corner-fileted flexure hinges is studied using several refinement stages of the matrix method to generate a model whose predictions are confirmed by finite element simulation.

1. Introduction

This work's focus is on planar serial flexure-based mechanisms and on their quasi-static response to actuation and external loading when both the load and the resulting displacements are in the mechanism's plane. The aim is to supplement the existing modeling algorithms with a method that is general with regards to mechanism structure, topology, loading, and boundary conditions, as well as in terms of the flexure hinge configurations and modeling refinement levels. The method can be implemented with relative ease to model, analyze, optimize, and design any specific application falling into this mechanism category.

Planar serial (or single-loop) flexure mechanisms are continuous chains formed of flexible links (particularly flexure hinges here) and rigid links that are rigidly interconnected, as pictured in Fig. 1.

The sequence of flexure hinges and rigid links of this mechanism can be covered continuously from one end of the mechanism (E_1 for instance) to the opposite end (E_2), so that a single loop is closed when connecting the end point E_2 back to the start point E_1 – as indicated by arrows in Fig. 1. The mechanism is assumed to be at least statically determined/constrained (with the understanding that it can also be over-constrained). Fig. 1 illustrates an over-constrained chain because its support points generate more than three reactions, which could be calculated using the three equilibrium equations available for planar statically-determined structures. A monolithically-built, statically-determined serial chain has only one link fixed which is generally at one end. Purely parallel and serial–parallel (hybrid) flexure-based mechanisms need at least two loops to cover all component links.

Serial flexure-based chains can be standalone mechanisms in robotic manipulators, (such as the free-fixed arm of Fig. 2(a)) or subsystems of more complex devices that are symmetric both structurally and in terms of loading (as detailed in the example analyzed later here). Serial chains can also be components of parallel or hybrid compliant mechanisms, as illustrated in Fig. 2(b), where three identical serial limbs connect to the output platform in a three-degree-of-freedom $xy\theta$ stage – see similar design presented in [1].

Planar flexible mechanisms with right circular flexure hinges have been studied extensively by means of kinetostatic or dynamic analytical models, specifically because analytical compliances have been provided since the 1950s in [2] for this particular flexure configuration. For instance, a five-bar precision-positioning mechanism with right circular flexures and rigid links was analyzed in [3]. Part of the mechanism studied in [3] is a two-stage planar serial displacement–amplification device whose output displacement is determined in terms of the input force of a piezoelectric block by only using the rotary bending and axial stiffnesses of

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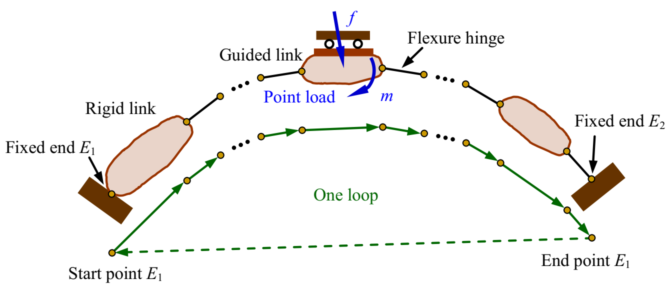

Fig. 1. Single-loop planar serial mechanism with flexure hinges and rigid links.

its flexure hinges. Single-loop mechanisms with regular rotation joints have been studied in [4] through an iterative inverse kinematic matrix method that includes the link deformations to correct the end effector's position. A device similar to that analyzed in [3] was studied in [5] based on the concepts of single- and multiple-loop mechanisms and considering the links connected by flexure hinges are flexible. The right circular flexure hinges were modeled as purely rotational springs (retaining only the rotary bending stiffness, which connects bending moment to slope) and the resulting quasi-static model predicted the output displacements in terms of the input displacements or forces by using the method of virtual work in a vector-matrix formulation. The simplest serial chain formed of a rigid link and a right circular flexure hinge in a lever arrangement was studied in [6], by using the axial stiffness and the translatory bending stiffness (which relates bending force to deflection) of the flexure hinge to predict the force–displacement relationship for both the basic chain and chains composed of multiple stages arranged in series. The stiffness of serial limbs with free-fixed ends and comprising three right circular flexure hinges interspersed with rigid links was formulated in [7] by means of the flexures' individual bending compliances. The model was used to formulate the overall quasi-static response of a double linear spring and of a three-degree-of-freedom spatial mechanism. A stiffness model for a planar serial chain composed of three rigid links and three right circular flexure hinges was derived in [8] by using the bending and axial compliances of the flexures. Three serial limbs were further incorporated into a parallel precision stage whose overall stiffness was evaluated based on the limbs' stiffnesses. The quasi-static behavior of planar and spatial serial compliant chains with generic flexure hinge configuration and various end boundary conditions was analyzed in [9], which took into consideration all compliances (related to bending, axial loading, shear, and torsion) by using a method based on Castigliano's theorem, as well as the loop-closure method. Two-stage amplification compliant limbs with right circular flexure hinges were also used in [10,11] to derive the stiffness of a parallel, three degree-of-freedom (DOF) $xy\theta$ stage. Similar planar $xy\theta$ stages with right circular flexure hinges under small deformations were modeled and analyzed in [12] – which utilized all six spatial compliances of the flexure, [1] – which derived a lumped-parameter dynamic model, [13,14] – which only used the rotary bending compliance of flexures, [15] – which employed finite element (FE) analysis to derive the six-DOF stiffness matrix of a flexure, or [16] – which used an FE model to generate a simplified, two-beam model of the flexure hinge. Displacement-amplification planar mechanisms were modeled in [17] – whose design incorporated right circular flexure hinges, [18–20] – where constant cross-section (leaf-spring) flexures were used to model the static or/and dynamic response, [21,22] – where right circularly corner-fileted flexure hinges and their planar compliances have been employed. Refs. [23,24] modeled and designed Scott–Russell mechanisms with right circular flexure hinges to be applied in micro-positioning and nano-manipulation. Other examples of modeling the static/dynamic response of mechanisms that incorporate planar serial flexure-based chains in more complex arrangements are those studied in [25] – a vertical-motion micro-positioning stage with right circular flexure hinges where only the rotary bending stiffness was considered, [26] – which designed

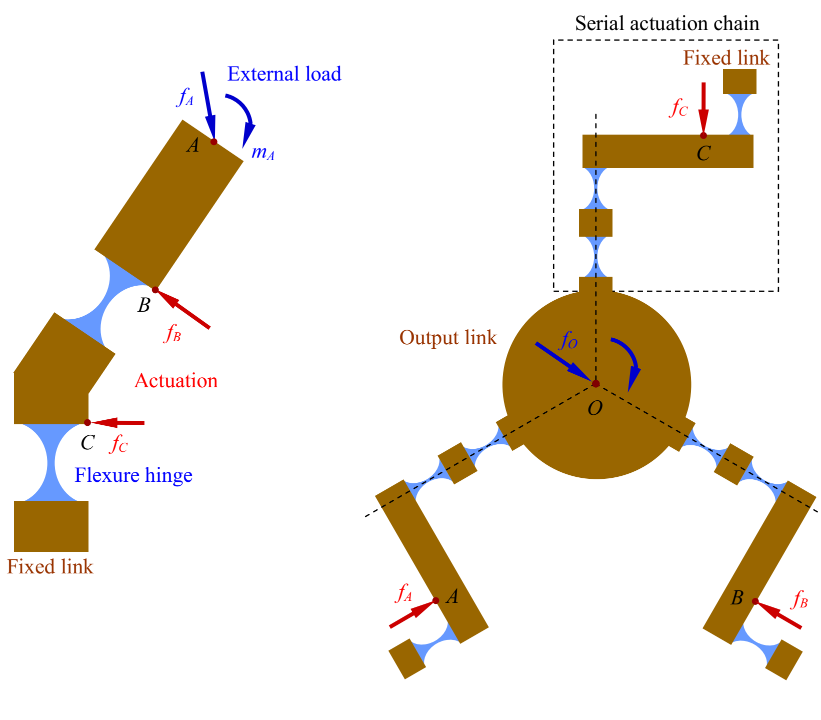

Fig. 2. Examples of planar serial mechanisms with flexure hinges: (a) robotic manipulator; (b) actuation limb as a triple-rotation-joint (*RRR*) chain in an $xy\theta$ stage – see [1].