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Fracture prediction for piezoelectric ceramics based on the electric field saturation concept

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Abstract

The electrical field saturation model is applied to the fracture prediction of piezoelectric materials containing electrically impermeable cracks. This model is analogously similar to the electric displacement saturation model that available in the literature. An electrical field saturation strip near the crack front is introduced in the analytical model. The stress intensity factor *K* and the energy release rate *G* are obtained in closed-form. It is found that fracture predictions based on *K* and *G* criteria are identical. Fracture predictions based on the electric field saturation model and the electric displacement model are also found to be the same.

Keywords: Piezoelectric materials; Cracks; Nonlinear fracture mechanics; Fracture prediction

1. Introduction

Piezoelectric ceramics are brittle and are used in the coupled electro-mechanical loading environments. It is important to study the fracture behavior of these advanced materials and to understand the influence of the electric field on their fracture strength. The fracture mechanics concept in traditional elasticity was expanded to piezoelectric ceramics (Suo et al., 1992). Several important issues regarding the effect of electric fields on crack growth in piezoelectric ceramics have been reported in the literature. Analysis based on linear piezoelectricity cannot explain the experimental observations that crack growth can be enhanced or retarded by applied electric fields. Hence, various non-linear models have been proposed. Among those

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models, the electrical displacement saturation model assumes that, under electromechanical loads, the region near the crack front is yield electrically to reach a saturation limit (Fulton and Gao, 1997; Gao et al., 1997; Ru, 1999; Wang, 2000).

On the other hand, when the electric field in a solid dielectric increases up to a certain value, the dielectric will breakdown (Kuffel and Zaengl, 1984; Suo, 1993). Therefore, it is reasonable to consider an electric field saturation strip near the crack tip. Recently, such a model has been proposed (Zhang and Gao, 2004). This paper establishes the analytical results based on this model. The stress intensity factor and the energy release rate for the electric field saturation model and the electric displacement saturation model are found to be different. However, the two models give the same predictions for the critical applied loads on the piezoelectric media.

2. Description of the problem

Investigated is an electrically insulated crack in an infinite piezoelectric medium, as shown in Fig. 1. Assume that all field variables are functions of x_1 and x_2 only. The coordinates x and y are coincident with x_1 and x_2 respectively. Constitutive equations for piezoelectric materials polarized along the x_2-direction subjected to mechanical and electrical fields can be written as

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ D_1 \\ D_2 \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{13} & 0 & 0 & -e_{31} \\ c_{13} & c_{33} & 0 & 0 & -e_{33} \\ 0 & 0 & c_{44} & -e_{15} & 0 \\ 0 & 0 & e_{15} & \in_{11} & 0 \\ e_{31} & e_{33} & 0 & 0 & \in_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \\ E_1 \\ E_2 \end{Bmatrix}, \tag{1}$$

where σ_{ij} and D_i are stresses and electric displacements, respectively; c_{ij},e_{ij}, and $\in_{ii}$ are elastic constants, piezoelectric constants, and dielectric permittivities, respectively. The strains ε_{ij} are related to the mechanical displacements u_i as $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$, where a comma indicates a partial derivative. The electric fields E_i are related to the electric potential ϕ as $E_i = -\phi_{,i}$.

In the absence of body forces and body charges, the governing equations can be written in terms of displacements u_1 and u_2 and the electric potential ϕ by inserting Eq. (1) into the mechanical and electrical equilibrium conditions

$$\sigma_{ji,j} = 0, \quad D_{i,i} = 0. \tag{2}$$

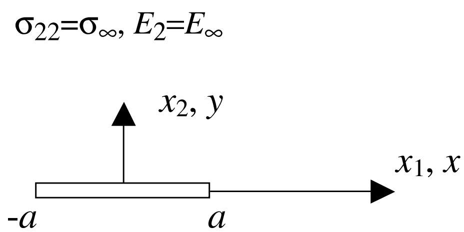

Fig. 1. A piezoelectric medium with a through-thickness crack.