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A steady line heat source in a decagonal quasicrystalline half-space

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Abstract

The problem of a steady line heat source within a decagonal quasicrystalline half-space, in which the boundary is traction-free, is investigated in detail by applying the complex variable technique. The line heat source is infinitely long in the period direction, then the plane strain condition prevails. The four complex stress functions are derived. It is found that the determination of the complex stress functions is independent of the thermal boundary conditions. The stress fields induced by the heat source are explicitly given based on the complex stress functions. Furthermore, we consider, both qualitatively and quantitatively, the distribution of σ_{11} on the boundary $y = 0$ of the half-space to demonstrate the influence of the phonon–phason coupling constant R on the stress field.

Keywords: Decagonal quasicrystal; Half-space; Heat source; Complex variable method

1. Introduction

Since the first observation of icosahedral quasicrystals in Al–Mn alloys by Shechtman et al. (1984), several other quasicrystals, such as the pentagonal, decagonal, dodecagonal and octagonal phases have been subsequently discovered. It has been experimentally verified that quasicrystals can really exist as stable phases, then it's necessary to take quasicrystals as a thermodynamic system and to establish the corresponding equilibrium thermodynamics. A comprehensive description of the equilibrium thermodynamics for quasicrystals can be found in the review article given by Hu et al. (1997). Thermal stress analyses for specific problems in quasicrystals are still very scant in the literature. Recently, Wang and Zhong (2003a) derived the general solution for plane strain problems in decagonal quasicrystals accounting for thermal effect. The obtained general solution was then employed to investigate the phonon and phason fields induced by a

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point heat source in an infinite decagonal quasicrystal, and thermal stresses due to disturbance of uniform heat flow by an insulated elliptical hole in an infinite decagonal quasicrystal. The solutions for the crack problems were obtained by setting the minor axis of the ellipse to be zero.

In this research, we investigate the phonon and phason fields induced by a steady line heat source within a decagonal quasicrystalline half-space. Here the line heat source is infinitely long in the period direction so that the plane strain condition is satisfied. Employing the complex variable method developed recently by us (see Wang and Zhong, 2003a in the presence of thermal effect, and Wang and Zhong, 2003b, 2004 in the absence of thermal effect), an exact solution to this problem is obtained. The four complex stress functions characterizing the phonon and phason stress fields are derived, and consequently the explicit expressions of the phonon and phason stress fields are presented. When the phonon–phason coupling constant R is taken to be zero, our results can just reduce to those for a steady heat source in an isotropic elastic half-space previously obtained by Wang (1988). To demonstrate the influence of the phonon–phason coupling constant R on the stress field, we consider, both qualitatively and quantitatively, the distribution of σ_{11} along the boundary $y = 0$ of the half-space. It is observed that the influence of R is apparent. For example, when $R = 0$, the maximum value of σ_{11} along the boundary always occurs at the point which is most proximate to the heat source; on the other hand, for some non-zero values of R, the maximum value of σ_{11} along the boundary can occur at a certain point which is further away from the heat source.

2. Complex variable formulation

For a decagonal quasicrystal whose period direction is the x_3-axis, and whose quasiperiodic plane is the (x_1, x_2)-plane, the basic equations, which take into account the thermal effects, are listed as follows (Hu et al., 1997).

In the absence of heat sources and body forces, the static equilibrium equations and heat conduction equation are

$$q_{i,i} = 0, \quad \sigma_{ij,j} = 0, \quad H_{ij,j} = 0. \tag{1}$$

The strain–displacement relations are

$$\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i}), \quad w_{ij} = w_{i,j}. \tag{2}$$

The linear constitutive equations take the form

$$q_1 = -k_{11}\theta_{,1}, \quad q_2 = -k_{11}\theta_{,2}, \quad q_3 = -k_{33}\theta_{,3}, \tag{3a}$$

$$\begin{aligned}
\sigma_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} + R(w_{11} + w_{22}) - \beta_1\theta, \\
\sigma_{22} &= C_{12}\varepsilon_{11} + C_{11}\varepsilon_{22} + C_{13}\varepsilon_{33} - R(w_{11} + w_{22}) - \beta_1\theta, \\
\sigma_{33} &= C_{13}\varepsilon_{11} + C_{13}\varepsilon_{22} + C_{33}\varepsilon_{33} - \beta_3\theta, \\
\sigma_{12} &= \sigma_{21} = (C_{11} - C_{12})\varepsilon_{12} - R(w_{12} - w_{21}), \\
\sigma_{23} &= \sigma_{32} = 2C_{44}\varepsilon_{23}, \\
\sigma_{13} &= \sigma_{31} = 2C_{44}\varepsilon_{13}, \\
H_{11} &= R(\varepsilon_{11} - \varepsilon_{22}) + K_1 w_{11} + K_2 w_{22}, \\
H_{22} &= R(\varepsilon_{11} - \varepsilon_{22}) + K_2 w_{11} + K_1 w_{22}, \\
H_{12} &= -2R\varepsilon_{12} + K_1 w_{12} - K_2 w_{21}, \\
H_{21} &= 2R\varepsilon_{12} - K_2 w_{12} + K_1 w_{21}, \\
H_{13} &= K_3 w_{13}, \\
H_{23} &= K_3 w_{23},
\end{aligned} \tag{3b}$$