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Semi-parametric proportional intensity models robustness for right-censored recurrent failure data

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Abstract

This paper reports the robustness of the four proportional intensity (PI) models: Prentice–Williams–Peterson-gap time (PWP-GT), PWP-total time (PWP-TT), Andersen–Gill (AG), and Wei–Lin–Weissfeld (WLW), for right-censored recurrent failure event data. The results are beneficial to practitioners in anticipating the more favorable engineering application domains and selecting appropriate PI models. The PWP-GT and AG prove to be models of choice over ranges of sample sizes, shape parameters, and censoring severity. At the smaller sample size ($U=60$), where there are 30 per class for a two-level covariate, the PWP-GT proves to perform well for moderate right-censoring ($P_c \leq 0.8$), where 80% of the units have some censoring, and moderately decreasing, constant, and moderately increasing rates of occurrence of failures (power-law NHPP shape parameter in the range of $0.8 \leq \delta \leq 1.8$). For the large sample size ($U=180$), the PWP-GT performs well for severe right-censoring ($0.8 < P_c \leq 1.0$), where 100% of the units have some censoring, and moderately decreasing, constant, and moderately increasing rates of occurrence of failures (power-law NHPP shape parameter in the range of $0.8 \leq \delta \leq 2.0$). The AG model proves to outperform the PWP-TT and WLW for stationary processes (HPP) across a wide range of right-censorship ($0.0 \leq P_c \leq 1.0$) and for sample sizes of 60 or more.

Keywords: Repairable systems reliability; Right-censoring; Recurrent events; Proportional intensity models; Non-homogeneous Poisson process; Prentice–Williams–Peterson

1. Introduction

Cox [1] proposed the distribution-free (semi-parametric) proportional hazards (PH) model in 1972 to account for covariate effects for single event failures (lifetime data) in a non-repairable system. The scope of this study focuses on recurring failure events in a repairable system. Failure time data on a repairable system are realizations of a stochastic point process, in which the instantaneous rate of occurrence of failures (ROCOF) is $\lambda(t)$ Prentice, Williams, and Peterson (PWP) [2] proposed a semi-parametric approach to model recurrent failure event data from a repairable system using two methods: PWP-GT (gap time) and PWP-TT (total time). Several researchers have subsequently proposed alternate modeling methods by modifying the risk set (common or event-specific baseline intensity function) and the risk interval (gap time, total time, or counting process). These include the Andersen–Gill (AG) [3] and Wei–Lin–Weissfeld (WLW) [4] models. The Cox-based PI regression models (PWP, AG, and WLW) extend the single-event PH models to deal with recurring events. These Cox-based regression models have been applied in medical studies (biostatistics field), such as recurrent infections in a patient.

Compared to the extensive literature on applications of the Cox-based PI regression models in the biostatistics field, there have been few reported engineering applications. Abundant federal funding received in biostatistics/medical research has advanced the PI models to become well developed and widely referenced. We propose that PI models for medical applications could also apply to recurring failure/repair data in engineering problems. The PWP-GT, PWP-TT, AG, and WLW models are potentially powerful analytical tools for engineering practitioners as they become better recognized and understood. Along these lines, Landers

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Nomenclature

Acronyms

AG	Andersen and Gill model
CI	confidence interval
DROCOF	decreasing rate of occurrence of failures
HPP	homogeneous Poisson process
IROCOF	increasing rate of occurrence of failures
i.i.d	independent and identically distributed
LWA	Lee, Wei, and Amato model
MTTF	mean time to failure
MAD	mean absolute deviation
MSE	mean squared error
NHPP	non-homogeneous Poisson process
PH	proportional hazards
PI	proportional intensity
PWP	Prentice, Williams, and Peterson model
PWP-GT	Prentice, Williams, and Peterson-gap time model
PWP-TT	Prentice, Williams, and Peterson-total time model
WLW	Wei, Lin, and Weissfeld model

Notation

$h(t;\mathbf{z})$	proportional hazard function
$h_0(t)$	baseline hazard function
i.i.d.	independent and identically distributed
N	successive failure count
$N(t)$	random variable for the number of failures in $(0,t]$; a counting process
n	an integer counting successive failure times; a stratification indicator subscript
P_c	censoring probability
T_1, T_2	the beginning and end of an event
T_n	random variable for cumulative time of occurrence of the nth failure
t_n	cumulative time of occurrence of the nth failure; a realization of T_n
U	sample size (number of units)
$Y_i^{(n)}$	an at-risk indicator in the AG model
$\mathbf{Z}(t)$	covariate process up to time t
$\mathbf{z}$	$(k\times 1)$ vector of covariates, $\mathbf{z}=(z_1,z_2,\ldots,z_k)'$
$\boldsymbol{\beta}_n$	$(k\times 1)$ vector of stratum-specific regression coefficients $\boldsymbol{\beta}=(\beta_1,\beta_2,\ldots,\beta_k)$
δ	shape parameter of a power-law NHPP
Δ	limit to time zero
λ_0	baseline value of λ for power-law NHPP
$\lambda_0(t)$	baseline intensity function
$\lambda_{0n}(t)$	stratum-specific baseline intensity function
$\lambda(t;\mathbf{z})$	proportional intensity function
υ	scale parameter of a power-law NHPP
υ_0	baseline value of υ, the scale parameter of a power-law NHPP
υ_1	alternate value of υ, the scale parameter of a power-law NHPP
$\wedge$	denotes an estimator
$'$	denotes the transpose of a vector

and Soroudi [5], Qureshi et al. [6], Vithala [7], and Landers et al. [8] have investigated robustness of the PWP-GT model, where the underlying recurrent failure time data are from a Non-homogeneous Poisson Process (NHPP) with a power-law or a log-linear intensity function. These references also report the engineering applications of the PWP-GT model cited in the literature (examples contained in Table 1).

Qureshi et al. [6] performed a robustness study to determine how well the PWP-GT method performed when applied to complete data from a failure process that was actually parametric (NHPP with power-law intensity function). They found that the PWP-GT model performs best for constant and moderately increasing rate of occurrence of failures (IROCOF) and decreasing rate of occurrence of failures (DROCOF) and for larger sample sizes from power-law NHPPs. Specifically, they concluded that for sample sizes of 60 (30 per class) or greater, the PWP-GT method is robust over the range of shape parameters $1.0\le\delta\le 3.0$, but tends to underestimate β for a DROCOF (e.g. BIAS $=-26\%$ at $\delta=0.5$) and overestimate β for an IROCOF (e.g. BIAS $=19\%$ at $\delta=3.0$). The true value of coefficient β lies within the 2σ confidence bounds on the estimate $\hat{\beta}$ for $1.0\le\delta\le 1.4$. Vithala [7] considered the case of log-linear increasing ROCOF, and concluded the PWP-GT model performs best for moderately increasing ROCOF and for larger sample sizes.

Both Qureshi et al. [6] and Vithala [7] have examined robustness of the PWP-GT model for complete (uncensored) data. However, the phenomenon of censoring is generally present in field data. Wei et al. [4] examined right-censoring data in a bladder cancer study, where the recurrence times of tumors for each patient were collected. Hu and Lawless [13] conducted a censoring experiment on automobile failure data to develop estimation procedures for measuring covariate effects.

This research has extended prior work to the important case of right-censorship and has examined other semi-parametric PI models (PWP-TT, AG, and WLW). This paper

Table 1
Engineering applications of proportional intensity models

Category	References
Marine gas turbine engines	Ascher [9]
Semiconductor industry	Ansell and Phillips [10]
Electrical industry	Ansell and Phillips [10]
Pipeline industry	Ansell and Phillips [10]
US army main battle tanks	Landers et al. [8]
Water supply industry	Ansell et al. [11,12]