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Minimum centroid neighborhood for minimum zone sphericity

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ABSTRACT

The minimum zone sphericity tolerance is derived from the ANSI and ISO standards for roundness and has extensive applications in the tribology of ball bearings, hip joints and other lubricated pairs. The worst-case proposed in this paper provides theoretical evidence that the minimum zone center of the two (circumscribed and inscribed reference) spheres with minimum radial separation containing the sampled spherical surface is included in a spherical neighborhood centered in the centroid of radius $2\pi^{-2}E_C$, where E_C is the sphericity error related to the centroid, which can be determined in closed form.

Such linear estimating (about 20% of E_C from the centroid, i.e., about one order of magnitude lower than the sphericity tolerance to be assessed) can be used to locate the sphere center with a given tolerance and as a search neighborhood for minimum zone center-based algorithms, such as metaheuristics (genetic algorithms, particle swarm optimization, etc.). The proposed upper bound has been experimentally assessed, using a genetic algorithm (GA) with parameters previously optimized for roundness and extended to three dimensions, which has overcome most of all available datasets from the literature that have been tested with center-based minimum zone algorithms by different authors. The optimum dataset size on artificially generated datasets is also discussed and it is speculated to allow the extension of the proposed upper bound to partial (or incomplete) spherical features.

1. Introduction

Sphericity affects the functional properties, e.g., tribology and lubrication, of various mechanical devices, such as ball bearings, hip joints and other spherical couplings and lubricated pairs.

According to ANSI [1], sphericity shares the same definition with circularity for form error evaluation from ISO [2]. The sphericity evaluation involves the determination of two concentric spheres, the circumscribed and inscribed (reference) spheres to the sampled points dataset (substitute feature), such that the radial separation between these two spheres is minimum (Fig. 1): minimum zone sphericity (MZS).

The evaluation of sphericity based on the minimum zone criterion is a non-linear and non-convex problem.

Some approaches to the MZS problem are based on the minimization of the minimum zone error E_{MZ} as a function of the minimum zone center C_{MZ}. The inconvenience is that this function has several local minima; consequently, the exploration is computationally intensive. The main purpose of current work is the definition and reduction of a search area for the C_{MZ}.

Some examples of center-based approaches are the simplex search/linear approximation [3,4] and metaheuristics like the particle swarm optimization (PSO) [5,6], ant systems [7], evolutionary [8] and genetic algorithms (GAs) [9–12].

Sphericity can be evaluated by roundness (or circularity) in different equatorial sections of the sphere surface.

Exact methods were proposed to evaluate sphericity based on the minimum zone criterion, such as [13], which is based on Voronoi tessellation, however this method is computationally intensive and it is not applicable to partial (or incomplete) spherical surfaces.

Fan and Lee [14] proposed an approach with minimum potential energy analogy to the minimum zone solution of spherical form error. The problem of finding the minimum zone sphericity error is transformed into that of finding the minimum elastic potential energy of the corresponding mechanical system. Chen and Liu [15] constructed three mathematical models to evaluate the minimum circumscribed sphere, the maximum inscribed sphere and the minimum zone sphere by directly resolving the simultaneous linear algebraic equations first. Then, the minimum zone solutions can be obtained using only five datapoints, which verify the 4-1, the 1-4, the 3-2 or the 2-3 condition.

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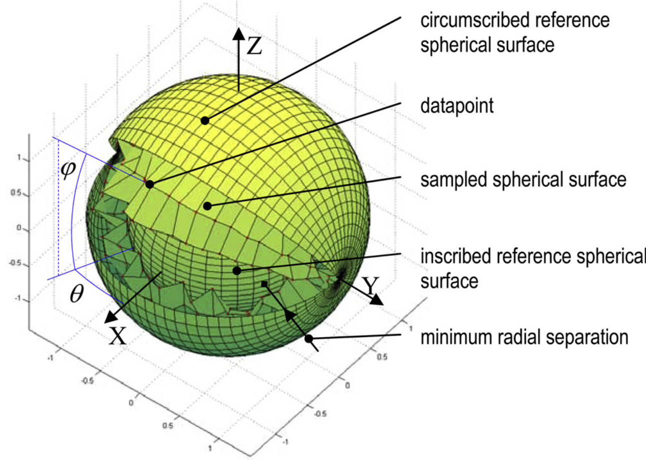

Fig. 1. The reference system and feature. The minimum zone sphericity error E_{MZ} is the minimum radial separation between the two concentric spherical surfaces respectively circumscribed and inscribed to the sampled dataset, centered in the (unknown) minimum zone center C_{MZ}.

Samuel and Shunmugam [16] established a minimum zone limaçon based on computational geometry to evaluate roundness error; with geometric methods, global optima are found by exhaustively checking every local minimum candidate.

Most of these methods process data point-by-point in the parameter space and move between datapoints by using some transition rules following the given nonlinear constraints one at a time. The point-to-point search method is sometime dangerous, because it is very possible to allocate false peaks in the multimodal search space for nonlinear optimization problems [17].

The main purpose of this work is to provide in closed form the minimum neighborhood of the centroid C that includes the minimum zone center C_{MZ} (Fig. 2). The definition of a restricted spherical volume centered in the centroid of the sampled sphere, which certainly includes the minimum zone center, has several applications: it can be used (i) *tout court* as a conservative first estimation of the minimum zone center position and of the minimum zone error; (ii) it may define a search neighborhood for a local search, e.g., by metaheuristics, such as genetic algorithms, particle swarm optimization, etc. By reducing the search area, the algorithm complexity and the computation time can be reduced [18]. Apart from [18], which uses a qualitatively adaptable search space size, in the literature only the following are available: [19] uses a fixed 1 mm search space size with a GA; [8] uses a fixed 2 mm search space size with an immune evolutionary algorithm; and later reduces the search space size to 0.1 mm with PSO [20]. All use the least squares center as the search space center. Fixed search space size poses, on one hand, the risk that the minimum zone center may not be included, particularly with partial features or non-uniformly distributed sampled points; on the other hand, too large search spaces increase the complexity and processing time.

2. Problem formulation

The MZT is the solution of the following optimization problem [11]:

$$\min\left[\max_{\theta_i=i\frac{2\pi}{n},\, i=1,\ldots,n,\, \varphi_j=j\frac{2\pi}{n},\, j=1,\ldots,\frac{n}{2}} s(x,y,z,\theta_i,\varphi_j)-\min_{\theta_i=i\frac{2\pi}{n},\, i=1,\ldots,n,\, \varphi_j=j\frac{2\pi}{n},\, j=1,\ldots\frac{n}{2}} s(x,y,z,\theta_i,\varphi_j)\right] \quad (1)$$

subject to $(x,y,z)\in S$

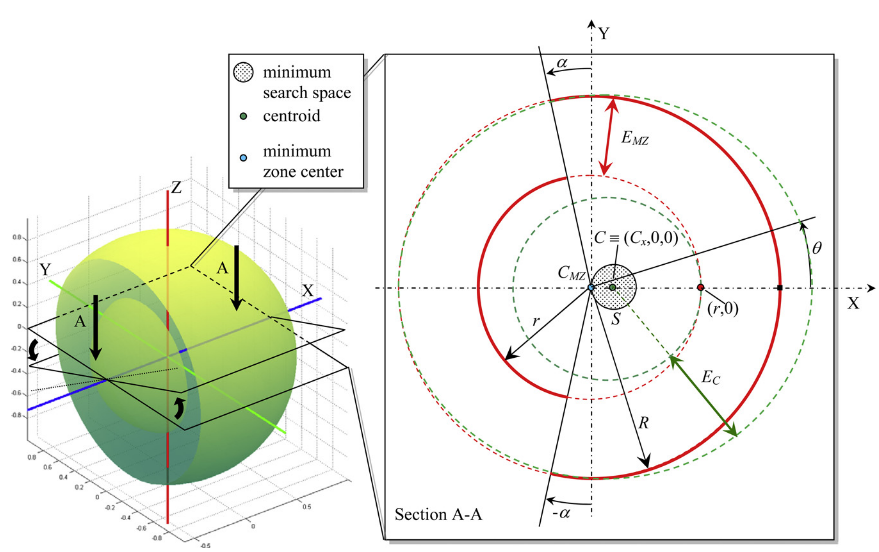

Fig. 2. Worst-case for the maximum distance (enhanced for clarity) between centroid C and minimum zone center C_{MZ}.