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Compliance modeling and analysis of statically indeterminate symmetric flexure structures

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ABSTRACT

The linear and angular compliance models for a class of statically indeterminate symmetric (SIS) flexure structures are established in this paper. Compared with a single flexure hinge, the SIS flexure structure is free of parasitic motions when a force or moment is applied. Thus, it can be treated as an ideal prismatic or revolute joint according to its load status. However, due to the inevitable axial tension, the load–deflection relationship of the SIS structure is nonlinear. Computational analyses are performed to investigate the influence of the axial tension. Computational results reveal that within small deflection range, the nonlinearity is very small and the axial tension can be neglected. In micro/nano scale applications, the motion range can be regarded as infinitesimal when compared with the dimension of the overall structure. Therefore, the influence of the axial tension would become negligible, and the analytical compliance models of the SIS structure are established using the integration of flexible beam. Compared with computational results, large modeling errors occur in the analytical models for the SIS structure with thick and short flexure hinges. Based on the observations from the error analyses, an error model is established and incorporated into the analytical compliance models to function as an error compensator. Utilizing the error compensator, the modeling accuracy of the compliance models can be improved, which is validated by the experimental results on a flexure-based mechanism.

1. Introduction

Flexure hinges have been widely used in the engineering applications where ultra precision positioning is of central importance [1,2]. Different from conventional mechanisms, flexure hinges transform motions based on their elastic deformations, which are free of friction, wear, backlash and lubrication. Therefore, flexure structures are capable of achieving high precision motions. The compliance/stiffness models for a single flexure hinge have been extensively investigated. Through the literature review, many different methods have been proposed, such as the integration of beam theory [3,4], Castigliano's second theorem [5–7], inverse conformal mapping [8], Pseudo Rigid Body (PRB) method [9], and empirical equations derived from finite element results [10–12]. The differences between these models have also been investigated in various situations [13]. In the mechanical design of flexure hinges, different types have been investigated, such as leaf-type [14], corner-fileted [15], V-shaped [16], conic sections with circular, elliptical, parabolic, and hyperbolic profile [5,17–21]. More complex types have also been proposed, such as the quadratic rational Bezier curves based unified geometric model proposed by Vallance el al. [22], and the three-segment flexure hinge proposed in [23].

Generally, a flexure hinge can be modeled as a revolute joint in micro/nano scale applications, and its angular deflection about *Z* axis is the primary motion. However, the flexure hinge suffers from cross-axis couplings. For a planar flexure hinge, there are strong couplings between the rotational and translational motions. Furthermore, the rotation center of a flexure hinge drifts whenever the hinge works, resulting in motion errors. In practice, multiple flexure hinges can be combined to form certain compliant structures to obtain decoupled characteristics [24,25]. Illustrated in Fig. 1(a), the parallelogram [26–29] has been widely used as the prismatic joint because it is capable of eliminating the rotational motions. However, parasitic motions in the transverse direction still exist and the asymmetric configuration leads to poor thermal performance. The double parallelogram [30,31], shown in Fig. 1(b), avoids parasitic motions in the parallelogram by introducing an intermediate linkage. However, the asymmetric configuration remains and the equivalent stiffness is reduced to half that of the parallelogram with identical parameters. Illustrated in Fig. 1(c), the symmetric parallelogram [32] is an equivalent structure of the double parallelogram, which retains the same stiffness. Due to the symmetry, this structure significantly reduces the thermally induced deformations. However, the off-axis stiffness is low and it is very weak in the

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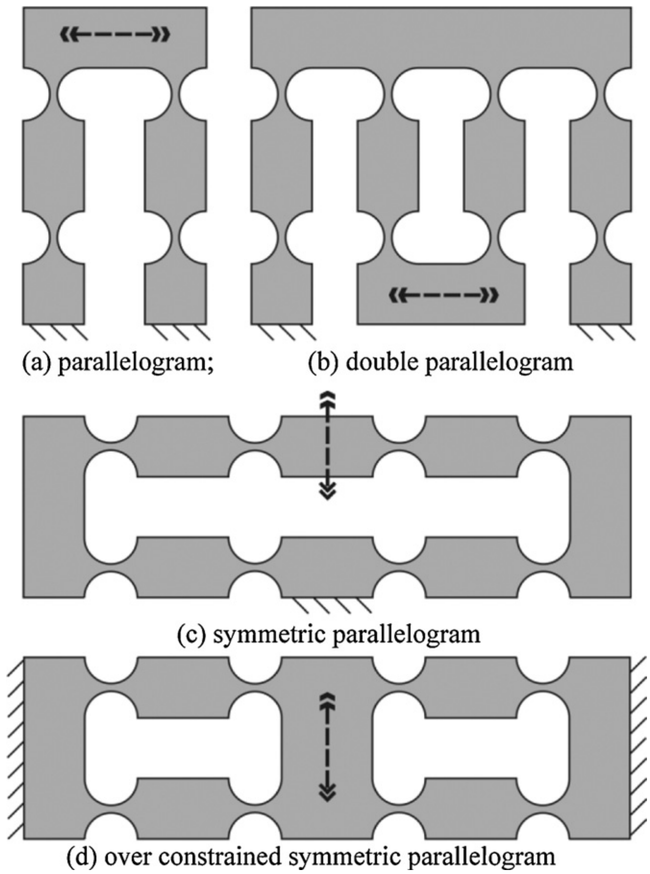

Fig. 1. Typical decoupled flexure structures. (Double headed arrows: working directions.)

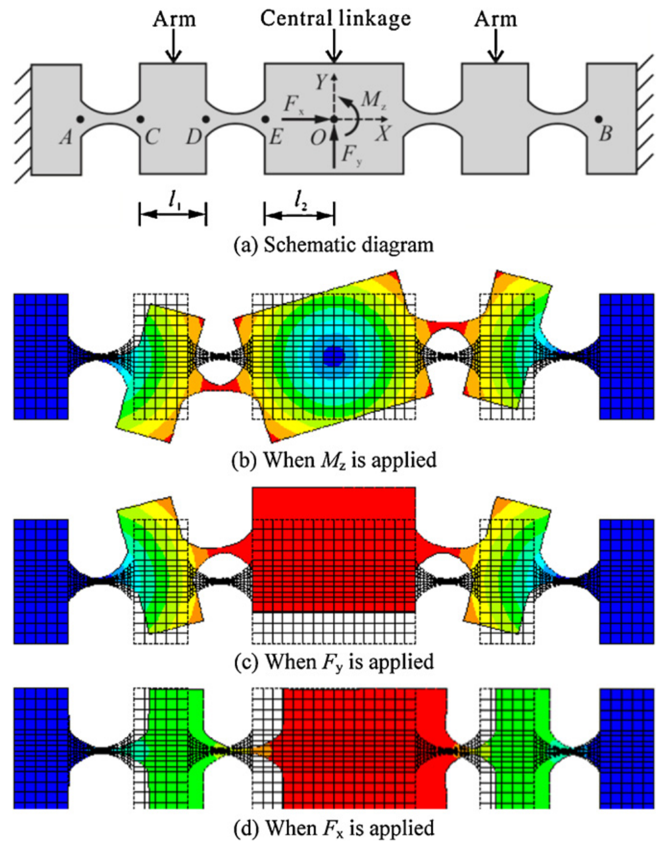

Fig. 2. Schematic diagram of the SIS flexure structure and its working principle.

transverse direction. Fig. 1(d) shows the diagram of an over-constrained symmetric parallelogram [25,33]. As both ends are clamped, the stiffness in the working and transverse directions can be significantly improved. However, the load-deflection relationship is nonlinear due to the axial tension. All the parallelogram-based structures shown in Fig. 1 are designated to function as prismatic joints, and thus they cannot be used in applications where rotations are required.

In the compliance/stiffness modeling of the flexure-based mechanism, the PRB method [9] and the Castigliano's second theorem [5,6] are the most commonly used methodologies. The PRB method provides an effective and simple way to estimate the mechanism's stiffness. It is particularly attractive in complex structures. However, the modeling error is inevitable if the axial and transverse deformations are not taken into consideration [34]. The Castigliano's second theorem derives the compliance of a given structure based on the stain energy. The deflection of a point caused by a given load is the partial derivative of the total strain energy with respect to the applied load.

As shown in Fig. 2, a class of statically indeterminate symmetric (SIS) flexure structures will be extensively investigated in this paper. In Fig. 2(a), a fundamental SIS flexure structure consists of one central linkage and two arms connected by four identical flexure hinges. The arm length is denoted as l_1 and the central linkage length is denoted as $2l_2$. The geometric parameters of the flexure hinge are defined in Fig. 3(a). Different from the aforementioned parallelogram-based structures, the SIS structure is symmetric about its center and both ends are clamped. As a result, it is more stable and the thermally induced deformations can be attenuated. Depending on the load status described in Fig. 2(a), the SIS structure exhibits distinct characteristics. Fig. 2(b) shows the deformation of the SIS structure when a moment about Z axis (M_z) is applied. Being symmetric, the rotation center does not drift. Therefore, the central linkage will rotate about the center point O, acting as an ideal revolute joint. In Fig. 2(c), if a force of F_y is applied in Y axis, the central linkage will translate in Y axis. In this case, the SIS structure can be regarded as half of the over-constrained symmetric parallelogram as shown in Fig. 1(d). The symmetry of the structure significantly attenuates the parasitic motions in X axis and the parasitic rotations about Z axis. When a force of F_x is applied in X axis, the central linkage will translate in X axis. Generally, the linear stiffness in X axis is much higher than its linear stiffness in Y axis. Therefore, in applications, the SIS structure can always be treated as rigid in X axis. Compared with the symmetric parallelogram in Fig. 1(c), the SIS structure has higher stability and higher off-axis stiffness, and in fact exhibits high degree of robustness to the transverse disturbance. Compared with the over-constrained symmetric parallelogram in Fig. 1(d), the SIS structure can also be used as a revolute joint. Therefore, the SIS structure in Fig. 2(a) is widely applicable in precision applications requiring high precision accuracy.

The SIS structure is not without disadvantages. By introducing the static indeterminacy, the axial tension exists whenever the SIS structure operates. In general, the load–deflection relationship of the SIS structure is nonlinear. However, through computational analyses in Section 3, it is demonstrated that within small deflection range, the nonlinearity is small enough and the influence of the axial tension can be neglected. Through literature review it is found that for such over-constrained flexure structures, the axial tension has always been neglected [6,25,35] as in micro/nano scale applications, the deflection range can always be regarded as infinitesimal. Subsequently, the compliance models of the SIS structure are derived from the integration of flexible beam based on the assumption that only the flexure hinges are flexible and