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Fracture mechanics of laminated glass subjected to blast loading

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Abstract

A failure criterion based on energy balance approach is introduced for the laminated glass panel subjected to blast loading. Based on this failure criterion, a damage factor is developed to assess the failure of the laminated glass panel. If the damage factor is less than one, the plate is safe otherwise unsafe. Trigonometric function is employed to express the transverse deflection and the Airy's stress function in von Karman's large deflection equations of a thin plate. The nonlinear ordinary differential equation of motion obtained using the Galerkin method is solved using Runge–Kutta method. The predicted results indicate that the breakages of the laminated glass may be caused by the negative phase of the blast load if the positive phase blast load is not violent enough to cause failure. Also, the size of glass shards the laminated glass plies breaks in to is predicted using the surface energy based failure model.

Keywords: Blast load; Laminated glass; Geometric nonlinearity; Linear viscoelasticity; Energy balance

1. Introduction

It is estimated that the majority of nonfatal injuries from bomb blasts result from airborne glass fragments from architectural glazing [1]. The use of laminated glass has been shown to mitigate this hazard.

The problem of window glass subjected to bomb explosion generally involves large deflections. The dynamic response of a monolithic glass plate subjected to a random fluctuating wind pressure was studied in Ref. [2]. They used von Karman nonlinear equations and solved it by a dynamic finite difference technique. The effects of negative phase of the blast load on the probability of monolithic glass panel failure was presented in [3] simplifying the plate to a linear spring-mass single-degree-of-freedom system. The damage probability of laminated glass plate subjected to blast loading was calculated in [4] using a two-parameter Weibull distribution. The effect of negative phase of blast loading on the dynamic responses of laminated glass panel

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subjected to blast loading was studied in [5] based on the closed-form solution from classical small deflection thin plate theory.

In this paper, a geometric nonlinear analytical solution of laminated glass panel subjected to blast loading is presented. Trigonometric functions are employed to express the transverse deflection and Airy's stress function in von Karman's large deflection equations of a thin plate. The Galerkin method is used to obtain the nonlinear ordinary differential equation of motion and then it is solved using Runge–Kutta method. A failure model based on Griffith energy balance is developed for the laminated glass panel subjected to a blast loading. The size of glass shards scattered from laminated glass plies is predicted using this model. The results of the failure model are compared with the available experimental results for the laminated glass subjected to blast loading.

2. The damage model

When a system is subjected to a load, it stores certain amount elastic strain energy. Based on the principle of the first law of thermodynamics, such a system will be in a nonequilibrium state. To return to an equilibrium state, the system has to release the stored energy. One such mechanism for energy release is the formation of a new crack or the growth of an existing crack. In such a process, the stored strain energy is conversed to surface energy [6,7]. Based on the Griffith energy balance criterion [8], the total potential energy of the system, U, may be written as [9]

$$U = U_0 - U_\mathrm{a} + U_\gamma, \tag{1}$$

where U_0 is elastic energy of the uncracked plate, U_a is decrease in the elastic energy caused by introduction of a crack in the plate, U_γ is increase in the surface energy caused by the formation of the crack surface. Since $U_\mathrm{a} = U_\mathrm{r} = 0$ before the formation of a crack, $U = U_0$. Therefore, Eq. (1), during the formation of a crack, will reduce to

$$U_\gamma = U_\mathrm{a}. \tag{2}$$

Eq. (2) shows that the surface energy needed to form crack surface equals to the elastic energy provided by the tensile stress in a glass ply. In the following sections, Eq. (2) is first applied to calculate the critical size of fragments that the laminated glass plate will break into under a blast. Then, a failure criterion will be proposed for a laminated glass panel subjected to blast load using Eq. (2).

2.1. Critical glass fragment size

Consider a fictional fragment with area $\Delta A = \Delta a \times \Delta b$ bound by cracks on all sides is shown in Fig. 1(a) located in the tensile side of a laminated glass panel. Assuming the fragment is formed after the glass ply shattered by tensile stresses. The free-body diagram of this fragment subjected to the tensile stresses is shown in Fig. 1(b). The surface energy is given by

$$U_\gamma = 2(\Delta a + \Delta b)h_\mathrm{g}\gamma_\mathrm{s}, \tag{3}$$

in which, γ_s is the surface energy of glass and h_g is the thickness of glass. $\gamma_\mathrm{s} = 3.9\ \mathrm{J/m^2}$ was reported in Ref. [6] for soda-lime glass in static state. The strain energy of the isolated fragment is given by

$$\begin{aligned} U_\mathrm{a} &= \frac{1}{2}\int\int_V\int (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy})\,\mathrm{d}V \\ &= \frac{1}{2E_\mathrm{g}}\int_x^{x+\Delta a}\int_y^{y+\Delta b}\int_{h_\mathrm{p}/2}^{h_\mathrm{g}+h_\mathrm{p}/2}\Big[(\sigma_x)^2 + (\sigma_y)^2 \\ &\quad + 2(1+v_\mathrm{g})(\tau_{xy})^2\Big]\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x, \end{aligned} \tag{4}$$

where h_p is the thickness of PVB interlayer E_g and v_g are the elastic modulus and the Poisson's ratio of glass. Integrating Eq. (4) using the stresses to be obtained later and then equating it to Eq. (3), the transcendental function is obtained as

$$f(x, y, t, \Delta a, \Delta b) = 0. \tag{5}$$

It is postulated that the fragment shape and size are some way related to plate geometry. To simplify the calculations, it is assumed that

$$\frac{\Delta b}{\Delta a} = \frac{b}{a} = r, \tag{6}$$

Eq. (5) now is reduces to

$$f(x, y, t, \Delta a) = 0. \tag{7}$$

Eq. (7) cannot be solved explicitly. Therefore, an iterative method is used to solve for Δa for a given