C. R. Mecanique 344 (2016) 502–509

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

Shercliff layers in strongly magnetic cylindrical Taylor–Couette flow

Couches de Shercliff dans un écoulement cylindrique fortement magnétique de Taylor–Couette

Rainer Hollerbach [a,*], Deborah Hulot [a,b]

[a] *Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, UK*
[b] *Institut national des sciences appliquées de Rouen, 76801 Saint-Étienne-du-Rouvray cedex, France*

ARTICLE INFO

Article history:
Received 28 October 2015
Accepted 29 February 2016
Available online 8 April 2016

Keywords:
Magnetohydrodynamics
Taylor–Couette flow

Mots-clés :
Magnétohydrodynamique
Écoulement de Taylor–Couette

ABSTRACT

We numerically compute the axisymmetric Taylor–Couette flow in the presence of axially periodic magnetic fields, with Hartmann numbers up to $Ha^2 = 10^7$. The geometry of the field singles out special field lines on which Shercliff layers form. These are simple shear layers for insulating boundaries, versus super-rotating or counter-rotating layers for conducting boundaries. Some field configurations have previously studied spherical analogs, but fundamentally new configurations also exist, having no spherical analogs. Finally, we explore the influence of azimuthal fields $B_\phi \sim r^{-1}\hat{\mathbf{e}}_\phi$ on these layers, and show that the flow is suppressed for conducting boundaries, but enhanced for insulating boundaries.

RÉSUMÉ

Nous modéliserons l'écoulement axisymétrique de Taylor–Couette en présence d'un champ magnétique axialement périodique, avec un nombre de Hartmann jusqu'à $Ha^2 = 10^7$. La géometrie du champ montre des lignes de champ sous forme de couche de Shercliff. On observe des couches de cisaillement lorsque les frontières sont isolantes, tandis que la rotation est excessive ou inversée pour les frontières conductrices. Certaines configurations de champs sont similaires à celles vues sous forme sphérique ; cependant, de nouvelles configurations existent. Enfin, nous découvrirons l'influence de champs azimutaux ($B_\phi \sim r^{-1}\hat{\mathbf{e}}_\phi$) sur ces couches et nous montrerons que l'écoulement diminue avec des bords conducteurs, alors qu'il s'accentue pour des frontières isolantes.

* Corresponding author.
E-mail address: rh@maths.leeds.ac.uk (R. Hollerbach).

http://dx.doi.org/10.1016/j.crme.2016.02.012

1. Introduction

Shercliff layers are free shear layers that can occur in the flow of an electrically conducting fluid when a sufficiently strong magnetic field is externally imposed [1]. They arise due to the strongly anisotropic nature of the Lorentz force, consisting of a tension along the magnetic field lines. The details of how the spatial structure of the imposed field overlaps with the geometry of the container can then single out special field lines on which Shercliff layers form.

For example, suppose we consider spherical Couette flow, the flow induced in a spherical shell where the inner sphere rotates and the outer one is fixed. Consider further two possible choices of magnetic fields to impose, a dipole $\mathbf{B}_d = 2\sigma^{-3}\cos\theta\,\hat{\mathbf{e}}_\sigma + \sigma^{-3}\sin\theta\,\hat{\mathbf{e}}_\theta$ and a uniform axial field $\mathbf{B}_a = \hat{\mathbf{e}}_z = \cos\theta\,\hat{\mathbf{e}}_\sigma - \sin\theta\,\hat{\mathbf{e}}_\theta$, where (σ, θ, ϕ) are standard spherical coordinates, and (z, r, ϕ) cylindrical coordinates. For the dipole field, there will be some field lines that link only to the inner sphere, and others that connect the two spheres. Similarly, for the axial field, there will be some field lines that link only to the outer sphere, and others that connect the two spheres. The tension in the field lines then ensures that any field lines linked to one boundary only are completely locked to that boundary, with the fluid either co-rotating with the inner sphere, or stationary together with the outer sphere. It is only on field lines that connect to both boundaries that the fluid is faced with conflicting conditions at the two ends of the line, and resolves this conflict by rotating at a rate intermediate between the two end values.

The entire domain is therefore naturally divided up into different regions, depending on how the field lines connect to the boundaries, with the angular velocity changing abruptly across those field lines separating different regions [2,3]. Furthermore, it is clear that there is nothing special about either the spherical geometry or these two particular fields. As long as both the container and the imposed field are axisymmetric, the same considerations will apply, and will always result in Shercliff layers forming on these special field lines where the linkage to the boundaries switches from one type to another. The thickness of these layers scales as $Ha^{-1/2}$, where the Hartmann number Ha is a measure of the strength of the imposed field [4].

Another intriguing result is the influence of the electromagnetic boundary conditions. The conclusion above, i.e. that Shercliff layers are simply shear layers on which the angular velocity switches to something intermediate between 0 at the outer boundary and 1 at the inner boundary, is valid only if both boundaries are insulating. If instead the inner sphere is conducting, a dipole field yields a so-called super-rotation, where the fluid within the Shercliff layer rotates faster than the inner sphere [2]. Alternatively, if the outer sphere is conducting, an axial field yields a counter-rotation, where the fluid within the Shercliff layer rotates in the opposite direction to the inner sphere [5]. In both of these cases, the degree of super-rotation or counter-rotation is around 20–30% of the inner sphere's rotation rate, independent of Ha (for sufficiently large values). Even more unexpected results are obtained if both boundaries are taken to be conducting; in this case, the degree of 'anomalous' rotation appears to increase indefinitely as Ha is increased in a numerical computation [5,6]. Various asymptotic analyses of this problem confirm that the anomalous rotation should be $O(1)$ if only one boundary is conducting, but $O(Ha^{1/2})$ if both boundaries are conducting [7–10].

Motivated by these counter-intuitive results, [11] performed a systematic investigation of linear combinations of dipole and axial fields, and showed that it is even possible to obtain both super-rotation and counter-rotation simultaneously. One finds easily enough that combinations of these two basic ingredients, dipole and axial, are sufficient to create all field line topologies that are possible in a spherical shell geometry. The purpose of this paper is to show that other topologies are possible in cylindrical geometry, and to numerically explore what happens in those cases. For example, we will show that it is possible to construct a field having a single field line that is tangent to both the inner and outer cylinders, with the tangency at the outer cylinder then suggesting a super-rotation, but the tangency at the inner cylinder suggesting a counter-rotation. So what does happen in that case? We will further explore what happens when azimuthal fields of the form $r^{-1}\hat{\mathbf{e}}_\phi$ are added, which also have no natural analog in spherical geometry.

Finally, it is worth noting that there have been several liquid metal experiments related to some of the topics considered here. These include spherical Couette flow in both dipole [12–14] and axial [15,16] fields, cylindrical Taylor–Couette flow in an axial field [17,18], and even electromagnetically driven flows [19,20]. However, inertia (finite Reynolds number) plays an important role in most of these results, unlike in the 'pure' Shercliff layer problem considered here. See also [21–27] for numerical results related to some of these experiments, as well as [28] for a general review of magnetohydrodynamic Couette flows.

2. Equations

We consider a cylindrical Taylor–Couette geometry with nondimensional radii $r_\mathrm{i} = 1$ and $r_\mathrm{o} = 2$. Periodicity is imposed in z, with a wavelength $z_0 = 4$. The precise choice $z_0 = 4$ is not crucial, with a broad range of $O(1)$ values yielding similar Shercliff layer structures. (Taking $z_0 \gg O(1)$ could well lead to different solutions though.)

In the inductionless limit, the nondimensional Navier–Stokes and magnetic induction equations are

$$\frac{\partial \mathbf{U}}{\partial t} = -\nabla p + \nabla^2\mathbf{U} - Re\,\mathbf{U}\cdot\nabla\mathbf{U} + Ha^2(\nabla\times\mathbf{b})\times\mathbf{B}_0 \tag{1}$$

$$\nabla^2\mathbf{b} = -\nabla\times(\mathbf{U}\times\mathbf{B}_0) \tag{2}$$