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Second-law analysis for buoyancy-driven hydromagnetic couple stress fluid flow through a porous channel

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ABSTRACT

This paper examines the combined effects of the buoyancy force and of the magnetic field on the entropy generation rate in the flow of a couple stress fluid through a porous vertical channel. The flow's dynamical equations were non-dimensionalised and solved via the application of the Adomian decomposition method (ADM). Variations of some thermo-physical parameters were conducted and discussed, with regard to the physics of the fluid. Our result shows that the entropy generation rate increases as the buoyancy increases in the fluid. In addition, the irreversibility in the flow system results mainly from the fluid's viscosity, ohmic heating, and the buoyancy.

1. Introduction

Over the last few decades, the study of the thermodynamic analysis involving channel fluid flows has attracted a lot of research effort due to its application in several renewable energy systems. Examples of this include the prediction of the efficiency of many thermal systems exchanging heat between two heat reservoirs including other Carnot systems. Basically, the process of energy generation usually culminates in the wastage of excessive energy, which is dissipated in the form of heat. Following from this, there is a need to minimise this inherent wastage by improving the energy of the system. In this regard, few research work had been reported in the literature [1–21]. Specifically, Adesanya and Makinde [1] reported the entropy generation in a couple stress fluid flowing steadily through a porous channel with slip at the isothermal walls, Das et al. [2] examined the entropy generation in a magnetohydrodynamic (MHD) pseudo-plastic nanofluid flow through a porous channel with convective heating. In addition, Adesanya and Makinde [3] studied the entropy generation rate in the couple stress fluid flowing through a porous channel with convective heating at the walls. Eegunjobi and Makinde [4] studied the effect of the buoyancy force and of the Navier slip on the entropy generation rate in a vertical porous channel. The authors also extended their work [5] by investigating the inherent irreversibility of heat transfer in the steady flow of a couple stress fluid through a vertical channel filled with porous materials.

In all the studies in Refs. [1–21] dealing with thermodynamic analysis linked with channel fluid flow, the combined effect of a uniform magnetic field applied transversely to the flow channel and the buoyancy force due to a change in the temperatures at the two boundary plates were neglected. However, magnetohydrodynamics (MHD) fluid flow, which is

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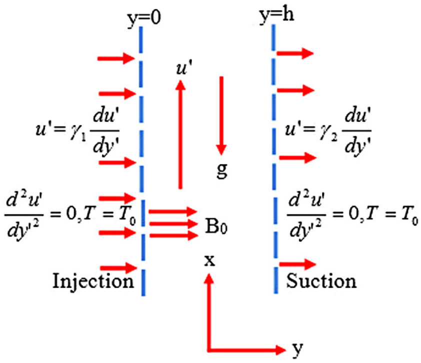

Fig. 1. The geometry of the model.

partly dependent on buoyancy, plays a vital role in many industrial and thermal engineering applications. For instance, it is useful in controlled irrigation systems, controlling extremely hot moving fluids like molten steel and liquid film, as well as in the polymer industry. Many researchers have examined the entropy analysis in a buoyancy-driven fluid flow. For example, Riley [22] presented a buoyancy-dependent MHD flow. Also, Alboussière et al. [23] investigated the asymptotic behaviour of a buoyancy-driven convection in the presence of a uniform magnetic field. Moreover, Eegunjobi and Makinde in [4,5] examined the second-law analysis for a buoyancy-driven incompressible fluid flow through a porous channel by imposing Navier slip conditions at the walls. In the same vein, Makinde and Chinyoka [24] examined the inherent irreversibility for flow and heat transfer inside a vertical channel made of two uniformly porous parallel plates with suction/injection under the combined action of a buoyancy force, a transverse magnetic field, and a constant pressure gradient.

Motivated by studies in [4,5,22–25], the objective of the present study is to examine the combined influence of the buoyancy and the magnetic field of the couple stress fluid on entropy generation within the flow channel, which has not been accounted for in the previous studies. The outcoming results are expected to enhance many industrial and thermal engineering processes whose working medium is a non-Newtonian fluid, with a view to minimise entropy generation, which tends to deplete the amount of useful energy for work.

To achieve this objective, flow-governing equations are formulated, non-dimensionalised, and approximate solutions to the dimensionless coupled non-linear boundary-value problem are obtained by using a semi-analytical Adomian decomposition method [26,27]. This method has been chosen because it does not require any linearisation, discretisation, use of initial guess or pertubation. These approximation solutions are used to compute the entropy generation rate and the irreversibility ratio.

In the following section, the problem is formulated, and a non-dimensional analysis is also presented. Section 3 contains the problem-solving method, the results are presented and discussed in Section 4, while Section 5 concludes the paper.

2. Mathematical formulation

A steady hydromagnetic non-Newtonian fluid flow between two permeable parallel vertical plates, with upthrust effect, is considered. The parallel plates are stationary regarding the motion of the fluid, as shown in the geometry of the problem (see Fig. 1). A 2-dimensional perpendicular coordinate system is employed, with the x-axis along the flow direction for the problem analysis. The width of the channel is $y = h$. Fluid injection occurs at the plate, where $y = 0$ at a uniform rate v_0. The system also allows fluid suction at the plate where $y = h$, at the same velocity v_0. A constant magnetic field of strength B_0 is applied perpendicular to the direction of the fluid flow. The flow problem is analysed so that no external voltage is applied to the flow system, and with negligible induced magnetic field and Hall effect. With reference to Boussinesq's approximation [28], a density difference exists in the flow system due to the temperature difference at the isothermal walls, which results in the buoyancy force contribution in the constitutive system equations. From this, the momentum and energy balance equations, with the local volumetric entropy generation rate (E_G) for the fluid flow, can be written as follows [1,4,12]:

$$v_0\,\rho\,\frac{\mathrm{d}u'}{\mathrm{d}y'} = -\frac{\mathrm{d}P}{\mathrm{d}x'} + \mu\frac{\mathrm{d}^2u'}{\mathrm{d}y'^2} - \eta\frac{\mathrm{d}^4u'}{\mathrm{d}y'^4} - \sigma_\mathrm{e}\,B_0^2\,u' + g\,\beta\,(T - T_0) \tag{1}$$

$$\rho\,c_P\,v_0\frac{\mathrm{d}T}{\mathrm{d}y'} = \kappa\frac{\mathrm{d}^2T}{\mathrm{d}y'^2} + \mu\Big(\frac{\mathrm{d}u'}{\mathrm{d}y'}\Big)^2 + \eta\Big(\frac{\mathrm{d}^2u'}{\mathrm{d}y'^2}\Big)^2 + \sigma_e B_0^2 u'^2 \tag{2}$$

$$E_G = \frac{\kappa}{T_0^2}\Big(\frac{\mathrm{d}T}{\mathrm{d}y'}\Big)^2 + \frac{\mu}{T_0}\Big(\frac{\mathrm{d}u'}{\mathrm{d}y'}\Big)^2 + \frac{\eta}{T_0}\Big(\frac{\mathrm{d}^2u'}{\mathrm{d}y'^2}\Big)^2 + \frac{\sigma_\mathrm{e}}{T_0}B_0^2u'^2 \tag{3}$$

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