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Thin hybrid linearly piezoelectric junctions

Les jonctions minces hybrides linéairement piézoélectriques

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ABSTRACT

We extend our previous study [1] devoted to thin linearly piezoelectric junctions to the case when the elastic, piezoelectric and dielectric coefficients of the junction are not of the same order of magnitude.

RÉSUMÉ

Nous étendons notre étude [1] consacrée aux jonctions minces linéairement piézoélectriques au cas où les coefficients élastiques, piézoélectriques et diélectriques de la jonction ne sont pas du même ordre de grandeur.

1. Introduction

Due to the wide range of values taken by the elastic, piezoelectric and dielectric coefficients of various devices, it is worthwhile to extend our previous study [1] devoted to thin linearly piezoelectric junctions to the case when the elastic, piezoelectric and dielectric coefficients of the junction *are not of the same order of magnitude*. Our various asymptotic models for a thin piezoelectric junction between two linearly piezoelectric or elastic bodies will be indexed by $p=(p_1,p_2,p_3)$ in $\{1,2,3,4\}^3$. Indices p_1 and p_2 are respectively relative to the magnitude of the elastic and dielectric coefficients of the adhesive with respect to that of the constant thickness 2ε of the layer containing the adhesive. More precisely, we assume that $h:=(\varepsilon,\mu)=(\varepsilon,\mu_{mm},\mu_{ee},\mu_{me})$ takes values in a countable set with a sole cluster point $\bar{h}\in\{0\}\times[0,+\infty]^3$, so that

$$\begin{cases} p_1=1: \ \bar{\mu}^1_{mm}:=\lim_{h\to\bar{h}}(2\varepsilon\mu_{mm})\in(0,+\infty) \\ p_1=2: \ \bar{\mu}^1_{mm}:=\lim_{h\to\bar{h}}(2\varepsilon\mu_{mm})=0 \\ \qquad\quad \bar{\mu}^2_{mm}:=\lim_{h\to\bar{h}}(\mu_{mm}/2\varepsilon)=+\infty \\ p_1=3: \ \bar{\mu}^2_{mm}:=\lim_{h\to\bar{h}}(\mu_{mm}/2\varepsilon)\in(0,+\infty) \\ p_1=4: \ \bar{\mu}^2_{mm}:=\lim_{h\to\bar{h}}(\mu_{mm}/2\varepsilon)=0 \end{cases} \qquad \begin{cases} p_2=1: \ \bar{\mu}^1_{ee}:=\lim_{h\to\bar{h}}(2\varepsilon\mu_{ee})\in(0,+\infty) \\ p_2=2: \ \bar{\mu}^1_{ee}:=\lim_{h\to\bar{h}}(2\varepsilon\mu_{ee})=0 \\ \qquad\quad \bar{\mu}^2_{ee}:=\lim_{h\to\bar{h}}(\mu_{ee}/2\varepsilon)=+\infty \\ p_2=3: \ \bar{\mu}^2_{ee}:=\lim_{h\to\bar{h}}(\mu_{ee}/2\varepsilon)\in(0,+\infty) \\ p_2=4: \ \bar{\mu}^2_{ee}:=\lim_{h\to\bar{h}}(\mu_{ee}/2\varepsilon)=0 \end{cases} \tag{1}$$

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The parameters μ_{mm}, μ_{ee}, μ_{me} respectively characterize the order of magnitude of the elastic, dielectric and piezoelectric coefficients of the adhesive. The case $p_1 = p_2$ being already treated in [1], in the following we assume $p_1 \neq p_2$. As in [1], index p_3 characterizes the status of the adherents but also that of the interfaces between adherents and adhesive:

$$\begin{cases} p_3 = 1: \text{ the two interfaces are electromechanically perfectly permeable} \\ p_3 = 2: \text{ the two interfaces are electrically permeable} \\ p_3 = 3: \text{ one interface is electrically permeable while the other one bears an electrode} \\ p_3 = 4: \text{ the two interfaces bear an electrode} \end{cases} \tag{2}$$

The physical situation is that of [1], which we recall as follows. Let Ω be a domain, with Lipschitz-continuous boundary, of $\mathbb{R}^3$, assimilated with the physical Euclidean space with basis $\{e_1, e_2, e_3\}$, whose intersection S with $\{x_3 = 0\}$ is a domain of $\mathbb{R}^2$ of positive two-dimensional Hausdorff measure $\mathcal{H}_2(S)$. Let $\Omega_\pm := \Omega \cap \{\pm x_3 > 0\}$ and ε be a small positive number, then adhesive and adherents occupy $B^\varepsilon := S \times (-\varepsilon, \varepsilon)$, $\Omega^\varepsilon_\pm := \Omega_\pm \pm \varepsilon e_3$, respectively; let $\Omega^\varepsilon = \Omega^\varepsilon_+ \cup \Omega^\varepsilon_-$, $S^\varepsilon_\pm := S \pm \varepsilon e_3$, $\mathcal{O}^\varepsilon := \Omega^\varepsilon \cup B^\varepsilon \cup_\pm S^\varepsilon_\pm$. Let $(\Gamma_{\mathrm{mD}}, \Gamma_{\mathrm{eD}})$, $(\Gamma_{\mathrm{eD}}, \Gamma_{\mathrm{eN}})$ be two partitions of $\partial\Omega$ with $\mathcal{H}_2(\Gamma_{\mathrm{mD}})$, $\mathcal{H}_2(\Gamma_{\mathrm{eD}}) > 0$ and $0 < \delta := \mathrm{dist}(\Gamma_{\mathrm{eD}}, S)$. For all Γ in $\{\Gamma_{\mathrm{mD}}, \Gamma_{\mathrm{mN}}, \Gamma_{\mathrm{eD}}, \Gamma_{\mathrm{eN}}\}$, $\Gamma_\pm$, $\Gamma^\varepsilon_\pm$, Γ^ε denotes $\Gamma \cap \{\pm x_3 > 0\}$, $\Gamma_\pm \pm \varepsilon e_3$, $\cup_\pm \Gamma^\varepsilon_\pm$, respectively; if $(\gamma_{\mathrm{D}}, \gamma_{\mathrm{N}})$ is a partition of $\gamma := \partial S$, we denote $\{\gamma_{\mathrm{D}}, \gamma_{\mathrm{N}}, \gamma\} \times (-\varepsilon, \varepsilon)$ by $\{\Gamma^\varepsilon_{\mathrm{DI}}, \Gamma^\varepsilon_{\mathrm{NI}}, \Gamma^\varepsilon_{\mathrm{lat}}\}$. The structure made of the adhesive and the two adherents, perfectly stuck together along $S^\varepsilon_\pm$, is clamped on $\Gamma^\varepsilon_{\mathrm{mD}}$ and subjected to body forces of density f^ε and to surface forces of density F^ε on $\Gamma^\varepsilon_{\mathrm{mD}}$ that vanishes on $\Gamma^\varepsilon_{\mathrm{lat}}$. Moreover, a given electric potential $\varphi^h_{p_0}$ is applied on $\Gamma^\varepsilon_{\mathrm{DI}}$ (and also on $\Gamma^\varepsilon_{\mathrm{eD}}$ when $p_3 = 1$), while electric charges of density d^ε appear on $\Gamma^\varepsilon_{\mathrm{NI}}$ (and also on $\Gamma^\varepsilon_{\mathrm{eN}}$ when $p_3 = 1$).

If σ^h_p, u^h_p, $e(u^h_p)$, D^h_p, φ^h_p respectively stand for the fields of stress, displacement, strain, electric displacement and electric potential, the constitutive equations of the structure, for all $\hat{p} := (p_1, p_2)$, read as:

$$\begin{cases} (\sigma^h_p, D^h_p) = M^\mu_{\mathrm{I}}\big(e(u^h_p), \nabla\varphi^h_p\big) & \text{in } B^\varepsilon\ \forall p_3 \in \{1,2,3,4\} \\ \begin{cases} (\sigma^h_p, D^h_p) = M^\varepsilon_{\mathrm{E}}\big(e(u^h_p), \nabla\varphi^h_p\big) & \text{in } \Omega^\varepsilon \text{ if } p_3 = 1 \\ \sigma^h_p = a^\varepsilon_{\mathrm{E}} e(u^h_p) & \text{in } \Omega^\varepsilon \text{ if } p_3 > 1 \end{cases} \end{cases} \tag{3}$$

where

$$(M^\varepsilon_{\mathrm{E}}, a^\varepsilon_{\mathrm{E}})(x) = (M_{\mathrm{E}}, a_{\mathrm{E}})(x \mp \varepsilon e_3) \quad \forall x \in \Omega^\varepsilon_\pm \tag{4}$$

$$\begin{cases} (M_{\mathrm{I}}, M_{\mathrm{E}}) \in L^\infty\big(S \times \Omega; \mathrm{Lin}(\mathbb{K})\big) \text{ such that} \\ M^\mu_{\mathrm{I}} := \begin{bmatrix} \mu_{mm} a_{\mathrm{I}} & -\mu_{me} b_{\mathrm{I}} \\ \mu_{me} b^T_{\mathrm{I}} & \mu_{ee} c_{\mathrm{I}} \end{bmatrix}, \quad M_{\mathrm{E}} := \begin{bmatrix} a_{\mathrm{E}} & -b_{\mathrm{E}} \\ b^T_{\mathrm{E}} & c_{\mathrm{E}} \end{bmatrix} \\ M_{\mathrm{P}} := \begin{bmatrix} a_{\mathrm{P}} & -b_{\mathrm{P}} \\ b^T_{\mathrm{P}} & c_{\mathrm{P}} \end{bmatrix}; \exists\kappa > 0 \quad \kappa|k|^2 \le M_{\mathrm{P}}(x)k \cdot k \quad \forall k \in \mathbb{K} := \mathbb{S}^3 \times \mathbb{R}^3 \text{ a.e. } x \in \Omega,\ \forall \mathrm{P} \in \{\mathrm{I}, \mathrm{E}\} \end{cases} \tag{5}$$

and $\mathrm{Lin}(\mathbb{K})$ is the space of linear operators on $\mathbb{K}$ whose inner product and norm are noted $\cdot$ and $|\cdot|$ as in $\mathbb{R}^3$ (the same notations for the norm and inner product also stand for $\mathbb{S}^N$ the space of $N \times N$ symmetric matrices).

Lastly we have to add the following conditions on $S^\varepsilon_\pm$:

$$\begin{cases} p_3 = 2 & D^h_p \cdot e_3 = 0 \quad \text{on } S^\varepsilon_\pm \\ p_3 = 3 & D^h_p \cdot e_3 = 0 \quad \text{on } S^\varepsilon_+, \quad \varphi^h_p = \varphi^h_{p_0} \text{ on } S^\varepsilon_- \\ p_3 = 4 & \varphi^h_p = \varphi^h_{p_0} \quad \text{on } S^\varepsilon_\pm \end{cases} \tag{6}$$

the electric potential $\varphi^h_{p_0}$ being given on S^ε_+ or $S^\varepsilon_\pm$.

It will be convenient to use the following notations:

$$\begin{cases} \hat{k} := (\hat{e}, \hat{g}) \quad \hat{e} := e_{\alpha\beta},\ 1 \le \alpha, \beta \le 2, \quad \hat{g} := (g_1, g_2), \quad \forall k = (e, g) \in \mathbb{K} \\ k(r) = k(v, \psi) := \big(e(v), \nabla\psi\big) \quad \forall r \in H^1(\mathcal{O}; \mathbb{R}^3 \times \mathbb{R}) \\ e(v) \in \mathcal{D}'(S; \mathbb{S}^2); \quad \big(e(v)\big)_{\alpha\beta} = \frac{1}{2}(\partial_\alpha v_\beta + \partial_\beta v_\alpha),\ 1 \le \alpha, \beta \le 2, \quad \forall v \in \mathcal{D}'(S; \mathbb{R}^3) \end{cases} \tag{7}$$

and the *same* symbol $e(\cdot)$ shall *also* stand for the symmetrized gradient in the sense of distributions of $\mathcal{D}'(\mathcal{O}; \mathbb{R}^3)$, $\mathcal{O} \in \{\mathcal{O}^\varepsilon, \Omega, \Omega \setminus S, B^\varepsilon, \Omega^\varepsilon\}$ or $\mathcal{D}'(S; \mathbb{R}^2)$. An electromechanical state with vanishing electric potential on $\Gamma^\varepsilon_{\mathrm{DI}}$ and on $\Gamma^\varepsilon_{\mathrm{eD}}$ when $p_3 = 1$ will belong to $V^\varepsilon_p := H^1_{\Gamma^\varepsilon_{\mathrm{mD}}}(\mathcal{O}^\varepsilon; \mathbb{R}^3) \times \Phi^\varepsilon_{p_3}$, with