ARTICLE IN PRESS
CRAS2B:3385

C. R. Mecanique ••• (••••) •••–•••

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

www.sciencedirect.com

A phase-resolved, depth-averaged non-hydrostatic numerical model for cross-shore wave propagation

Xinhua Lu [a,*], Xiaofeng Zhang [a], Bing Mao [b], Bingjiang Dong [c]

[a] *State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan 430072, China*
[b] *Yangtze River Scientific Research Institute, Wuhan 430015, China*
[c] *Hydrology Bureau, Yangtze River Water Resource Commission, Wuhan 430010, China*

ARTICLE INFO

Article history:
Received 26 June 2015
Accepted 14 September 2015
Available online xxxx

Keywords:
Non-hydrostatic
Dispersivity
Wave breaking
Well-balanced
Shock-capturing
HLL

ABSTRACT

In this study, a phase-resolved and depth-averaged non-hydrostatic numerical model (SNH model) is developed. The non-incremental pressure-correction method is employed to solve the equation system in two successive steps. Firstly, an approximate Riemann solver in the framework of finite volume methods is employed to solve the hydrostatic shallow-water equations (SWE) on a collocated grid to obtain provisional solutions. Then, the intermediate solutions is updated by considering the non-hydrostatic pressure effect; a semi-staggered grid is used in this step to avoid predicting checkboard pressure field. A series of benchmark tests are used to validate the numerical model, showing that the developed model is well-balanced and describes the wetting and drying processes accurately. By employing a shock-capturing numerical scheme, the wave-breaking phenomenon is reasonably simulated without using any ad-hoc techniques. Compared with the SWE model, the wave shape can be well-preserved and the numerical predictions are much improved by using the SNH model.

1. Introduction

The surf-zone and inner-shelf regions are of paramount importance for human beings. Accurate prediction of the flow motions in these regions is vital for designing and protecting engineering projects and is the basis for building early tsunami warning systems [1,2]. Besides, the prediction of flow features in these regions is important for studying coastal sediment transport and the associated beach profile changes [3–5].

For modeling the cross-shore hydrodynamics, phase-resolved and depth-averaged models are widely adopted. These models include the ones solving the nonlinear shallow-water equations (SWE), the Boussinesq-type equations (BTE) and the depth-averaged non-hydrostatic shallow-water equations (SNH). Of all the three types of models, the recently proposed SNH model framework [6] seems to be very promising. Firstly, compared with the SWE, which does not account for the wave dispersion effect, the SNH can represent weak wave dispersivity and thus is suitable for modeling long wave as well as short wave motions. Secondly, to simulate the wave-breaking phenomenon, which is commonly seen in the nearshore area, the BTE model generally requires an ad-hoc technique to numerically dissipate the wave energy during the wave-breaking process; in this regard, an artificial viscosity term [7,8] or a surface roller model [9,10], or a local switch from resolving the BTE to SWE in the vicinity of the breaking wave fronts [11–13], is widely used. Compared with a BTE model, the SNH

* Corresponding author.
E-mail address: xhluhh@whu.edu.cn (X. Lu).

http://dx.doi.org/10.1016/j.crme.2015.09.005

model has the potential to handle the wave-breaking process well when a shock-capturing numerical scheme is employed. Besides, the BTE contains high-order spatial derivatives that are difficult for numerical implementations, while for the SNH, the order of derivatives is generally one (when the diffusion terms are omitted) and thus easier for numerical realizations.

To date, very few SNH models have been developed and, among them, most are developed using the finite-difference method [6,14] or the finite-element method [15,16]. In viewing that the finite-volume Godunov-type schemes devised based on the hyperbolic conservation laws generally intrinsically have the shock-capturing property, in this study, we aim to develop a SNH model based on the finite-volume Godunov-type scheme to adapt to wave-breaking simulations without using any ad-hoc techniques. Also, the well-balanced property (e.g., the C-property) for the existing SNH models is generally not verified and in this work the developed SNH model will achieve this property. Note that it is important for a numerical scheme to satisfy the well-balanced requirement to accurately predict the wave phase speeds and the flow motions involving wetting and drying processes [17,18].

The rest of the paper is organized as follows. Section 2 presents the governing equations and the numerical algorithm for the SNH model. The specially designed grid layout and the essentials for the numerical discretizations are demonstrated. In Section 3, numerical tests are used to verify the various properties of the numerical model. Finally, conclusions and discussions are given in Section 4.

2. Numerical model

2.1. Governing equations

By depth-integrating the vertical 2D Reynolds-averaged Navier–Stokes equations and using the following kinematic boundary conditions at the free surface and water bottom

$$w_\mathrm{s} = \frac{\partial z}{\partial t} + u_\mathrm{s}\frac{\partial z}{\partial x} \tag{1}$$

$$w_\mathrm{b} = u_\mathrm{b}\frac{\partial z_\mathrm{b}}{\partial x} \tag{2}$$

the governing equations for the 1D SNH model can be derived as follows [6]

$$\frac{\partial z}{\partial t} + \frac{\partial q_x}{\partial x} = 0 \tag{3}$$

$$\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x}\left(\bar{u}q_x + \frac{1}{2}gh^2\right) = -gh\frac{\partial z_\mathrm{b}}{\partial x} - \frac{gn^2 q_x|q_x|}{h^{7/3}} - \frac{h}{2\rho}\frac{\partial \Gamma_\mathrm{b}}{\partial x} - \frac{\Gamma_\mathrm{b}}{2\rho}\frac{\partial (z+z_\mathrm{b})}{\partial x} \tag{4}$$

$$\frac{\partial \overline{w}}{\partial t} = \frac{\Gamma_\mathrm{b}}{\rho h} \tag{5}$$

Here, t denotes time; x denotes the horizontal coordinate in the cross-shore direction; z and z_b respectively denote the water surface and bed elevations above a horizontal reference level; u and w are the velocity components in the horizontal and vertical directions; the subscripts 's' and 'b' denote the values at the free surface and water bottom, respectively; $h = z - z_\mathrm{b}$ denotes the water depth; $q_x = \bar{u}\cdot h$ is the discharge per unit width with $\overline{(\cdot)}$ denoting the depth-averaging operator; Γ_b denotes the non-hydrostatic pressure at the water bottom; $\rho = 1000\ \mathrm{kg/m^3}$ is the density of water; g is the acceleration due to gravity; n is the Manning's roughness coefficient. To be remarked that in the derivations, the non-hydrostatic pressure and the vertical velocity are assumed to vary linearly with water depth [6]; the shear stress at the water bottom (i.e., the bed friction force) is closed by the second term on the right-hand side of Eq. (4). For brevity, the overbar of variables in Eqs. (4) and (5) will be dropped hereafter.

2.2. Numerical algorithm

The non-incremental pressure-correction method is employed for solving the governing equations, which divides the solution into two successive steps, namely, the hydrostatic and non-hydrostatic steps.

2.2.1. Hydrostatic step

In the hydrostatic step, the classic SWE is solved, which in a conservative vector form can be written as

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = \boldsymbol{S} \tag{6}$$

where the vectors are defined by

$$\boldsymbol{U} = \begin{bmatrix} z \\ q_x \end{bmatrix},\ \boldsymbol{F} = \begin{bmatrix} q_x \\ uq_x + \frac{1}{2}gh^2 \end{bmatrix},\ \boldsymbol{S} = \boldsymbol{S}_0 + \boldsymbol{S}_f = \begin{bmatrix} 0 \\ -gh\frac{\partial z_\mathrm{b}}{\partial x} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{gn^2 q_x|q_x|}{h^{7/3}} \end{bmatrix} \tag{7}$$