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An objective perspective for classic flow classification criteria

Une perspective objective pour des critères classiques de classification d'écoulements

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ABSTRACT

Four classic criteria used to the classification of complex flows are discussed here. These criteria are useful to identify regions of the flow related to shear, elongation or rigid-body motion. These usual criteria, namely Q, Δ, λ_2 and $\lambda_{cr}/\lambda_{ci}$, use the fluid's rate-of-rotation tensor, which is known to vary with respect to a reference frame. The advantages of using objective (invariant with respect to a general transformation on the reference frame) criteria are discussed in the present work. In this connection, we construct versions of classic criteria replacing standard vorticity, a non-objective quantity, by effective vorticity, a rate-of-rotation tensor with respect to the angular velocity of the eigenvectors of the strain-rate tensor. The classic criteria and their corresponding objective versions are applied to classify two complex flows: the transient ABC flow and the flow through the abrupt 4:1 contraction. It is shown that the objective versions of the criteria provide richer information on the kinematics of the flow.

RÉSUMÉ

Quatre critères classiques utilisés pour la classification des écoulements complexes sont étudiés ici. Ces critères sont utiles pour identifier les régions de l'écoulement liées au cisaillement, à l'extention ou au mouvement de corps rigides. Ces critères habituels, à savoir Q, Δ, λ_2 and $\lambda_{cr}/\lambda_{ci}$, utilisent le tenseur taux de rotation du fluide, qui est connu pour varier par rapport au système de référence. Les avantages qu'il y a à utiliser des critères objectifs (invariants par rapport à une transformation générale pour un système de référence) sont discutés dans le present travail. À cet égard, nous construisons des versions des critères classiques en remplaçant la vorticité standard, une quantité non objective, par la vorticité effective, un taux de rotation par rapport à la vitesse angulaire des vecteurs propres du tenseur taux de déformation. Les critères

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classiques et leurs versions objectives correspondantes sont appliqués pour classifier deux écoulements complexes : l'écoulement ABC transitoire et l'écoulement à travers une contraction brusque 4:1. Les versions objectives de ces critères fournissent des informations plus riches pour la cinématique de l'écoulement.

1. Introduction

In Fluid Mechanics, flow visualisation is an important subject, since fundamental aspects of the flow can be captured by observation. Post-processing Computational Fluid Dynamics (CFD) is also a field that makes important contributions to the understanding of the flow. Complex flows exhibit different kinds of motion that depend on position and time. In these flows, it is common to find swirling motions in different parts of the domain. In order to locate and visualise these regions, a criterion of vortex identification is generally used to see the manifestation of the rotational character of the flow.

However, the concept of *vortex* is still a cause of dissension within the scientific community. As a consequence, there are several criteria available in the literature that are used to identify rotational structures in the flow. In other words, there is no quantity, in the mathematical sense, that is consensually accepted in the literature as a definition of a vortex. Some of the non-consensual issues that are raised in this context are the question of whether the vortex is an Eulerian or a Lagrangian entity, and of whether it should be defined on a kinematic or in a dynamical basis.

Comparisons among the different criteria are still a subject of investigation (e.g., [1]). An important point raised by Haller [2] is the requirement that a vortex should be an Euclidean invariant entity, i.e. invariant under arbitrary changes of the reference frame. This requirement affects the vortex concept, since, before that work, only the Galilean invariance was invoked to define a vortex [3]. We can stress here that the classic vortex definitions, such as the Q-criterion by Hunt et al. [4], the Δ-criterion by Chong et al. [5], the λ_2-criterion by Jeong and Hussain [3], and the $\lambda_{cr}/\lambda_{ci}$-criterion by Chakraborty et al. [6], enjoy only Galilean invariance (i.e. they are invariant to constant velocity translating frames).

The arguments to adopt an objective criterion for vortex identification are the following. First, if one observer identifies a certain region as being a vortex while, for another one, this region is not a vortex, there is no reason to privilege the verdict stated by one observer with respect to the other. Secondly, we have to keep in mind the advantages of building a criterion for vortex identification. One clear purpose of identifying a region as being a vortex is to connect the rotational character of the flow with another phenomenon besides the flow itself. It is consensual that processes like: the convection in a heat transfer problem, the degree of mixture of different fluids, the percentage of components due chemical reaction in a flow, the intensity of polymer stretching due to the flow, and other transport phenomena problems cannot be observer-dependent. Hence, if a vortex is non-objective, the logic of cause-effect that could link the flow character with one of these measurers of the intensity that a certain phenomenon is occurring is weaker, when compared to an objective criterion.

In the present work, we employ objective versions of four classic criteria largely used in the literature. The classic criteria and their respective objective versions are analysed and applied for two benchmark cases, the transient Arnold–Beltrami–Childress (ABC) flow [7–9] and the flow through a 4:1 contraction.

2. Classic criteria

In the following, we briefly present four criteria that are currently used in the literature to classify different regions of the flow. These criteria are Eulerian and Galilean-invariant and were recently selected by Pierce et al. [1] to evaluate, for instance, boundary layer flows.

The Q-criterion was proposed by Hunt et al. [4] in the context of incompressible flows. Besides local pressure minima, they required that, to identify a vortex, the second invariant of the velocity gradient tensor, $\nabla \boldsymbol{u}$ (defined by $\nabla \boldsymbol{u} = (\partial u_j/\partial x_i)\boldsymbol{e}_i\boldsymbol{e}_j$, where $\boldsymbol{u} = u_i\boldsymbol{e}_i$ is the velocity vector field), should be positive. This condition can be expressed for incompressible flows as a function of the Euclidean norms[1] of the symmetric, $\boldsymbol{D} = (\boldsymbol{L} + \boldsymbol{L}^{\mathrm{T}})/2$, and skew symmetric, $\boldsymbol{W} = (\boldsymbol{L} - \boldsymbol{L}^{\mathrm{T}})/2$, parts of the velocity gradient (where $\boldsymbol{L} = (\partial u_i/\partial x_j)\boldsymbol{e}_i\boldsymbol{e}_j$ is the transpose of $\nabla \boldsymbol{u}$). The condition for the Q-criterion can be expressed as follows:

$$Q = \frac{1}{2}\left(\|\boldsymbol{W}\|^2 - \|\boldsymbol{D}\|^2\right) > 0 \tag{1}$$

The Δ-criterion proposed by Chong et al. [5] is based on the assumption of an equivalence between a vortex and complex eigenvalues of the velocity gradient tensor.[2] Complex eigenvalues of the velocity gradient are a sign of vorticity dominance with respect to rate of strain, since the symmetric rate-of-strain tensor can only have real eigenvalues. Mathematically, the Δ-criterion can be defined as

[1] The Euclidean norm of a generic second-order tensor $\boldsymbol{A}$ is $\|\boldsymbol{A}\| = \sqrt{tr(\boldsymbol{A}\cdot\boldsymbol{A}^{\mathrm{T}})}$, where $tr(\cdot)$ is the first invariant (trace) of a given second-order tensor.

[2] The expression *velocity gradient* is used interchangeably for $\boldsymbol{L}$ or $\nabla \boldsymbol{u}$.