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On mode localisation in tensile plate buckling

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Abstract

In some cases, a tensile load may induce small compressive stresses and buckling phenomenon. In this Note, we present an analytical study of the buckling of long stretched plates. From this analytical model, one explains the wavelength selection and establishes that the in-plane stress localisation induces the mode localisation. Comparison with numerical results shows a good agreement. ***To cite this article: N. Jacques, M. Potier-Ferry, C. R. Mecanique 333 (2005).***

Résumé

Flambement localisé de bandes sous traction. Cet article porte sur le flambement de bandes longues induit par des contraintes compressives secondaires apparaissant lors de l'application d'un chargement de traction. Un modèle analytique a été mis au point. Il permet d'expliquer la forme caractéristique des modes lors de ce type de flambement. Elle est principalement due à la localisation des contraintes compressives et à l'existence d'un mécanisme de sélection des longueurs d'onde. Ces résultats sont validés par des calculs par éléments finis. ***Pour citer cet article : N. Jacques, M. Potier-Ferry, C. R. Mecanique 333 (2005).***

Keywords: Solids and structures; Buckling; Thin plates; Wavelength selection; Mode localisation

Mots-clés : Solides et structures ; Flambement localisé ; Plaques minces ; Sélection des longueurs d'ondes

1. Introduction

The buckling of structures due to compressive loads is a well-known problem. A tensile loading can also induce buckling. Indeed, for some structures, application of a tensile load leads to the occurrence of compressive stresses generally perpendicular to the main loading direction. The most famous example is the Yoshida buckling test [1], where a square plate is subjected to a diagonal tension inducing an inhomogeneous stress field. Another example is proposed by Timoshenko and Goodier [2], who study a rectangular plate subjected to a parabolic traction on two opposite sides. They observe the occurrence of transverse compressive stresses. Friedl et al. [3] consider the buckling of stretched strips with special boundary conditions that constrain the lateral displacements on the loaded edges and

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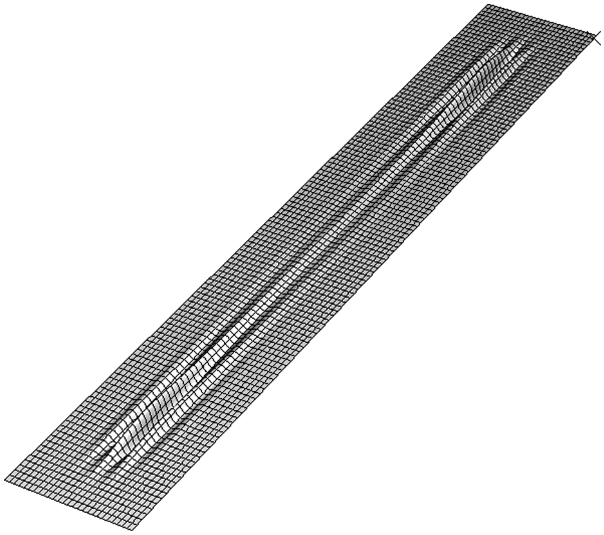

Fig. 1. Buckling of a stretched strip, from [3]. The load is applied in the longitudinal direction, the lateral displacements are prevented on the short edges and the long edges are free. The aspect ratio is seven.

Fig. 1. Flambage d'une plaque sous traction, d'après [3]. Le chargement est appliqué selon la longueur, les conditions aux limites empêchent tout déplacement latéral au niveau des cotés les plus courts, les autres bords sont libres. La longueur de la plaque est égale à sept fois sa largeur.

lead to transverse compressive stresses near these edges. Fig. 1 shows a buckling mode obtained by Friedl et al. [3] with use of a finite element analysis. Long waves are present near the centreline and the displacement is larger in the area where compressive stresses reach their maximum value. Far from the loaded edges, where the transverse stresses are vanished, buckles decay.

In this paper, we propose an analytical study of the buckling of long plates under global tension. We point out that the buckling mode localisation is induced by the localisation of the in-plane compressive stresses.

2. Governing equations

We considerer a rectangular plate of thickness t, width B and length L, with $t \ll B < L$. x and y denote the in-plane coordinates, x in the longitudinal direction, y in the transverse direction such that the domain is $0 \leqslant x \leqslant L$, $-B/2 \leqslant y \leqslant B/2$. We assume an elastic isotropic behaviour with E the Young's modulus and ν the Poisson's ratio. A tensile load is applied on the short lateral edges. Since inhomogeneous loading or special boundary conditions are considered, a complex membrane force field (1) appears. In the framework of a linear buckling analysis, this pre-buckling stress field (1) is assumed to be proportional to the applied traction.

$$\big[N(x,y)\big] = \begin{bmatrix} N_x(x,y) & N_{xy}(x,y) \\ N_{xy}(x,y) & N_y(x,y) \end{bmatrix} \tag{1}$$

If the length L is sufficiently large, the Saint-Venant's principle implies that, far from the loaded edges, the traction becomes uniform, i.e. only the longitudinal membrane forces N_x are non-zero and are constant through the width. So, transverse forces are located near the short edges.

It is well known that bifurcation occurs when the quadratic part of the potential energy P_2 changes sign, i.e. $P_2 = 0$ and $\delta P_2 = 0$. In the context of a thin plate model, the quadratic part of the potential energy reduces to:

$$2P_2 = \int_S \left(D(\Delta w)^2 + N_x \left(\frac{\partial w}{\partial x}\right)^2 + N_y \left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy} \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \right) \mathrm{d}x\,\mathrm{d}y \tag{2}$$

where $w(x,y)$ is the out-of-plane displacement. D is the bending stiffness modulus (3).

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{3}$$