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Fluid–solid interactions: modeling, simulation, bio-mechanical applications

A two-dimensional effective model describing fluid–structure interaction in blood flow: analysis, simulation and experimental validation

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Abstract

We derive a closed system of effective equations describing a time-dependent flow of a viscous incompressible Newtonian fluid through a long and narrow elastic tube. The 3D axially symmetric incompressible Navier–Stokes equations are used to model the flow. Two models are used to describe the tube wall: the linear membrane shell model and the linearly elastic membrane and the curved, linearly elastic Koiter shell model. We study the behavior of the coupled fluid–structure interaction problem in the limit when the ratio between the radius and the length of the tube, ε, tends to zero. We obtain the reduced equations that are of Biot type with memory. An interesting feature of the reduced equations is that the memory term explicitly captures the viscoelastic nature of the coupled problem. Our model provides significant improvement over the standard 1D approximations of the fluid–structure interaction problem, all of which assume an ad hoc closure assumption for the velocity profile. We performed experimental validation of the reduced model using a mock circulatory flow loop assembled at the Cardiovascular Research Laboratory at the Texas Heart Institute. Experimental results show excellent agreement with the numerically calculated solution. Major applications include blood flow through large human arteries. ***To cite this article: S. Čanić et al., C. R. Mecanique 333 (2005).***

Résumé

Un modèle efficace bidimensionnel décrivant l'interaction fluide–structure dans l'écoulement sanguin : analyse, simulation et validation expérimentale. Nous obtenons un système fermé d'équations efficaces, décrivant l'écoulement non-stationnaire d'un fluide newtonien incompressible visqueux à travers un tuyau élastique long et de faible épaisseur. Pour modéliser l'écoulement, nous utilisons le système de Navier–Stokes 3D axisymétrique et incompressible. Deux modèles sont employés pour décrire le comportement élastique de la paroi latérale : les équations de Navier pour une membrane courbe élastique linéaire, et ensuite le modèle de Koiter, d'une coque courbe, élastique linéaire. Nous étudions le comportement du système lorsque le rapport ε, entre l'épaisseur caractéristique et la longueur du tube, tend vers zéro. Nous obtenons les équations efficaces, essentiellement 1D, qui sont du type de Biot avec mémoire. Une caractéristique intéressante des équations efficaces est que le terme de mémoire capture explicitement la nature viscoélastique du problème couplé. Notre modèle efficace fournit une amélioration significative par rapport

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aux modèles 1D standards de l'interaction fluide–structure, qui nécessitent une formule de fermeture pour la vitesse, proposée ad hoc. Nous avons effectué la validation expérimentale du modèle réduit en utilisant la boucle d'écoulement simulé au Cardiovascular Research Laboratory, Texas Heart Institute. Les résultats expérimentaux montrent un accord excellent avec la solution calculée numériquement. L'application principale inclut l'écoulement sanguin à travers les grandes artères du corps humain. ***Pour citer cet article : S. Čanić et al., C. R. Mecanique 333 (2005).***

Keywords: Computational fluid mechanics; Blood flow; Asymptotic methods; Fluid–structure interaction; 3D Navier–Stokes equations

Mots-clés : Mécanique des fluides numérique ; Écoulements sanguin ; Méthodes asymptotiques ; Interaction fluide–structure ; Système de Navier–Stokes 3D

1. Introduction

This work is motivated by the study of blood flow in compliant arteries. In medium to large vessels such as the human aorta and iliac arteries, blood can be modeled as a viscous, incompressible Newtonian fluid, [1,2]. Driven by a time-periodic pressure pulse caused by the contractions and relaxations of the heart muscle, blood flow interacts with the pulsation of arteries. Modeling and simulation of the fluid–structure interaction between blood flow and arterial walls have been studied by many authors, see, for example, [3–8,1]. However, real-time calculations of large sections of the vascular system are still out of reach. Simplified models need to be used whenever possible. In axially symmetric sections of the vascular system one-dimensional models have been used to speed up the simulation, [9–11,5, 12,8,1]. These models have two drawbacks: they are not closed (an ad hoc assumption needs to be made on the shape of the axial velocity profile to close the system) and outflow boundary conditions generate nonphysiological reflected waves that contaminate the flow. The latter is due to the fact that the system is hyperbolic and Dirichlet boundary conditions give rise to the reflections from the artificially posed outlet boundary that are of the same magnitude as the physiological waves themselves, see [13,5]. In the present article we derive a simplified, effective model that gets around both drawbacks. The resulting equations are closed (the closure follows from the three-dimensional problem itself), and the nonphysiological reflected waves are minimized by the fact that the model equations are of mixed hyperbolic-parabolic type, with memory. The memory terms explicitly capture the observed viscoelastic nature of the fluid–structure interaction in blood flow. Although the resulting equations are two-dimensional, their simplified form allows a decomposition into a set of coupled one-dimensional problems, thereby allowing numerical simulation with complexity of the one-dimensional problems. In this article we present the derivation of the effective equations, a numerical method for their simulation and experimental validation performed on a mock flow loop at the Cardiovascular Research Laboratory at the Texas Heart Institute. The experimental validation shows excellent agreement with the numerically calculated solution.

2. The three-dimensional fluid–structure interaction model

We study the flow of an incompressible, viscous Newtonian fluid through a cylinder with compliant walls. In the reference state the cylinder is $L > 0$ units long and $2R > 0$ units wide. The aspect ratio $\varepsilon := R/L > 0$ is assumed to be small. For a given $R, L > 0$ denote the reference cylinder by $\Omega_\varepsilon = \{(r\cos\theta, r\sin\theta, z) \in \mathbb{R}^3\colon\ r \in (0, R), \theta \in (0, 2\pi), z \in (0, L)\}$ and its lateral boundary by $\Sigma_\varepsilon = \{(R\cos\theta, R\sin\theta, z) \in \mathbb{R}^3\colon\ \theta \in (0, 2\pi), z \in (0, L)\}$; see Fig. 1. We study a time-dependent flow driven by the time-dependent inlet and outlet boundary data. The compliant cylinder and its boundary deforms as a result of the fluid–structure interaction between the fluid occupying the domain and the cylinder's boundary.

We assume that the lateral wall of the cylinder behaves as a homogeneous, isentropic, linearly elastic shell of thickness h. We consider two linearly elastic shell models: the linearly elastic membrane model (1) and the linear Koiter shell model (2), studied in [14–17]. Accounting for only radial displacements $\eta^\varepsilon(z, t)$ and assuming a prestressed reference configuration at reference pressure p_{ref} [18,19], the model equations, in Lagrangian coordinates, take the following form: