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Fluid–solid interactions: modeling, simulation, bio-mechanical applications

Direct simulation of the motion of neutrally buoyant balls in a three-dimensional Poiseuille flow

Tsorng-Whay Pan *, Roland Glowinski

University of Houston, Department of Mathematics, Houston, TX 77204, USA

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Abstract

In a previous article the authors introduced a Lagrange multiplier based fictitious domain method. Their goal in the present article is to apply a generalization of the above method to: (i) the numerical simulation of the motion of neutrally buoyant particles in a three-dimensional Poiseuille flow; (ii) study – via direct numerical simulations – the migration of neutrally buoyant balls in the tube Poiseuille flow of an incompressible Newtonian viscous fluid. Simulations made with one and several particles show that, as expected, the Segré–Silberberg effect takes place. ***To cite this article: T.-W. Pan, R. Glowinski, C. R. Mecanique 333 (2005).***

Résumé

Simulation directe du mouvement de particules sphériques de flottabilité neutre dans un écoulement de Poiseuille tri-dimensionnel. Dans un autre article, les auteurs ont introduit une méthode de domaine fictif avec multiplicateurs de Lagrange. Leur objectif dans le présent article est d'appliquer une généralisation de la méthode ci-dessus à : (i) la simulation numérique du mouvement de particules interagissant avec un écoulement de Poiseuille tri-dimensionnel lorsque fluide et particules ont la même densité ; (ii) l'étude – par simulation numérique directe – de la migration de particules sphériques interagissant avec l'écoulement de Poisseuille, dans un tube de section ciculaire, d'un fluide Newtonien, visqueux, incompressible, de même densité que les particules. Comme prévu, ces simulations, effectuées avec une ou plusieurs particules, mettent en evidence l'effet de Segré–Silberberg. ***Pour citer cet article : T.-W. Pan, R. Glowinski, C. R. Mecanique 333 (2005).***

Keywords: Computational fluid mechanics; Particulate flow; Solid–liquid flow; Neutrally buoyant particles; Fictitious domain methods; Distributed Lagrange multipliers; Operator-splitting methods; Finite element methods; Segré–Silberberg effect

Mots-clés : Mécanique des fluides numérique ; Ecoulements particulaires ; Ecoulements solide–liquide ; Particules de flottabilité neutre ; Méthodes de domaines fictifs ; Multiplicateurs de Lagrange distribués ; Méthodes de décomposition d'opérateurs ; Méthodes d'éléments finis ; Effet de Segré–Silberberg

1. Introduction

The problem of particle motions in shear flows is crucially important in many engineering areas, such as the handling of fluid–solid mixtures in slurries, colloids, and fluidized beds. The experiments of Segré and Silberberg [1,2]

* Corresponding author.
E-mail address: pan@math.uh.edu (T.-W. Pan).

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have had a large influence on fluid mechanics studies of migration and lift of particles. They studied the migration of dilute suspensions of neutrally buoyant spheres in a pipe flow at Reynolds numbers between 2 and 700. The particles migrate away from the wall and centerline and accumulate at about 0.6 of the pipe radius from the centerline. Karnis el al. [3] verified the same phenomenon and observed that particles migrate faster for larger flow rate, closer to the wall for larger flow rate and closer to the axis for larger rigid spheres. The 'anomalous' motion observed is attributed to the nonlinear effect of inertia. Comprehensive reviews of experimental and theoretical works have been given by Brenner [4], Cox and Mason [5], Leal [6], Feuillebois [7], and McLaughlin [8] among others.

Direct numerical simulations have been used for understanding particle motion in shear flows. Feng el al. [9] investigated the motion of neutrally buoyant and non-neutrally buoyant circular particles in plane Couette and Poiseuille flows using a finite element method and obtained qualitative agreement with the results of perturbation theories and of experiments. Inamuro et al. [10] used the lattice Boltzmann method to study the motions of neutrally buoyant circular disks in a pressure driven plane Poiseuille flow. The Segré–Silberberg effect was found. They found that the equilibrium position of the particle is closer to the wall as the Reynolds number increases from about 12 to 96; but moves away from the wall as either the diameter of disk or the length of the channel is increased. Pan and Glowinski have generalized the *distributed Lagrange multiplier/fictitious domain method* (DLM/FD) for the numerical simulation of particulate flow (see [11–13]) to the case where the particles are neutrally buoyant in [14] for two-dimensional flows and confirmed via simulations that the phenomenon of collisions between particles is one of the key factors driving particles to the central region of the plane Poiseuille flow. Concerning three-dimensional computational results, Yang et al. [15] have recently studied the migration of a neutrally buoyant ball in a tube Poiseuille flow by using an arbitrary Lagrangian–Eulerian moving mesh technique.

In this article, we have extended the methodology in [14] to three-dimensional flows and performed simulations of the migration of neutrally buoyant balls in a tube Poiseuille flow. The content of this article is as follows: in Section 2, we discuss a fictitious domain formulation of the model problem concerning the case of the neutrally buoyant balls moving freely in a three-dimensional Poiseuille flow; then in Section 3 we discuss briefly the time and space discretization issues, and in Section 4 we present and comment the results of numerical experiments involving one and five neutrally buoyant balls.

2. A fictitious domain formulation of the model problem

All the fluid–solid interactions to be considered in this article concern the flow of fluid–solid particle mixtures in a cylindrical tube (denoted by **T** in the sequel) with a circular cross-section. In order to take a full advantage of the fictitious domain approach we will embed **T** in a cylindrical tube (denoted by Ω) with a square cross section whose edge length is equal to the diameter of the **T** cross-section. We will start our discussion with a one particle situation. Therefore, let $\Omega \subset \mathbb{R}^3$ be a rectangular parallelepiped. We suppose that Ω is filled with a *Newtonian incompressible viscous fluid* (of *density* ρ_f and *viscosity* μ_f) and that it contains a moving neutrally buoyant rigid particle B centered at $\mathbf{G} = \{G_1, G_2, G_3\}^t$ of *density* ρ_f, as shown in Fig. 1, which shows also the inclusion in Ω of the cylinder **T** mentioned above; we suppose that the central axis of both cylinders is parallel to the x_3-axis. The flow is modeled by the *Navier–Stokes equations* while the particle motion is described by the *Euler–Newton's equations*. We introduce (with $\mathrm{d}\mathbf{x} = \mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x_3$) the following functional spaces:

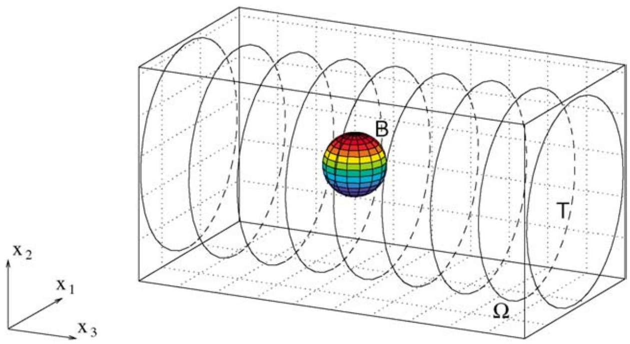

Fig. 1. An example of three-dimensional flow region with one rigid body.