Available online at www.sciencedirect.com
SCIENCE DIRECT
C. R. Mecanique 333 (2005) 923–933
COMPTES RENDUS MECANIQUE
http://france.elsevier.com/direct/CRAS2B/

Fluid–solid interactions: modeling, simulation, bio-mechanical applications

Apparent viscosity of a mixture of a Newtonian fluid and interacting particles

Aline Lefebvre, Bertrand Maury *

Laboratoire de mathématiques, université Paris-Sud, 91405 Orsay cedex, France

Available online 17 November 2005

Abstract

We investigate the behavior of fluid–particle mixtures subject to shear stress, by mean of direct simulation. This approach is meant to give some hints to explain the influence of interacting red cells on the global behavior of the blood. We concentrate on the apparent viscosity, which we define as a macroscopic quantity which characterizes the resistance of a mixture against externally imposed shear motion. Our main purpose is to explain the non-monotonous variations of this apparent viscosity when a mixture of fluid and interacting particles is submitted to shear stress during a certain time interval. Our analysis of these variations is based on preliminary theoretical remarks, and some computations for some well-chosen static configurations. ***To cite this article: A. Lefebvre, B. Maury, C. R. Mecanique 333 (2005).***

Résumé

Viscosité apparente d'un mélange de fluide Newtonien et de particules en interaction. Nous présentons une étude du comportement global d'un mélange de fluide newtonien et de particules rigides par la simulation directe. Cette approche apporte des éléments d'analyse de l'influence d'inclusions rigides en interaction (comme dans le cas des globules rouges dans le sang) sur le comportement global du mélange complexe. Nous nous sommes concentrés ici sur la viscosité apparente, que nous définissons comme une quantité macroscopique qui caractérise la résistance d'un fluide complexe à un mouvement de cisaillement imposé. Notre objectif principal est d'expliquer les variations non monotones de cette viscosité apparente au cours du temps, lorsque les particules interagissent. Notre analyse se base sur des remarques théoriques préliminaires et sur un certain nombre de calculs de cette viscosité pour des configurations représentatives. ***Pour citer cet article : A. Lefebvre, B. Maury, C. R. Mecanique 333 (2005).***

Keywords: Computational fluid mechanics; Apparent viscosity; Fluid–particle flow; Finite element method; Arbitrary Lagrangian Eulerian method

Mots-clés : Mécanique des fluides numérique ; Viscosité apparente ; Écoulements fluide–particles ; Méthode des Éléments Finis ; Méthode Arbitraire Lagrange–Euler

1. Introduction

The viscosity of a diluted suspension can be estimated from the exact solution to the Stokes equations for a single particle in an infinite fluid domain (this approach dates back to Einstein [1] in 1906). In many situations, e.g. red

* Corresponding author.
E-mail addresses: Aline.Lefebvre@math.u-psud.fr (A. Lefebvre), Bertrand.Maury@math.u-psud.fr (B. Maury).

1631-0721/$ – see front matter
doi:10.1016/j.crme.2005.10.007

cells in the blood, the dilution assumption is no longer valid, and inclusions are likely to interact with each other in a complex way. Under some assumptions the behaviour of neighbouring particles can be described by mean of analytic expressions or asymptotic development. See e.g. [2] where the motion of two spheres in a shear flow is described. More recently, Stokesian Dynamics has been applied to compute the motion of spheres in a linear Stokes flow for particular geometries (see [3,4]). But the overall behaviour of nonhomogeneous, many-body mixtures under general shear conditions calls for the use of direct numerical methods.

As a consequence, direct simulation of fluid–particle mixtures motivated a great amount of research during the last decade. Some authors, like Glowinski (see [5,6]) use a Cartesian mesh which covers the overall computational domain, and the rigid motion is taken into account by a Lagrange multiplier. The other class of approaches relies on a moving mesh which follows the geometry of the fluid domain. This approach has been followed by [7–9].

The present work is based on this second approach, which we think is more adapted to shear flow of highly concentrated suspension, as the presence of a (possibly) fine mesh which covers interparticle gaps makes it possible to compute accurately in these high stress regions. Moreover, integrating the degrees of freedom for the particles into the finite element space allows to compute highly viscous flows with unconditional stability, whereas other methods which decouple fluid and particles are more adapted to situations where inertia (at least inertia of the rigid bodies) plays a significant role. Although the numerical approach we follow would make it possible to handle Navier–Stokes flows and general geometries, we chose to limit ourselves here to Stokes flow, in order to show how a globally nonlinear behaviour can be recovered even though the instantaneous fluid model is itself linear.

2. Continuous model

We consider a rectangular domain Ω filled with a mixture of a Newtonian fluid and N rigid particles. The viscosity of the fluid is denoted by μ. All particles are circular, and their common radius is r. We denote by $\mathbf{F}_i$ the force exerted on particle i. We shall consider the situation where $\mathbf{F}_i$ is a sum of forces exerted by the other particles. The mixture domain Ω is a rectangle $2a \times L$ (see Fig. 1). The flow is periodic in the y-direction. Left and right walls are supposed to move vertically with velocities $-U_0\mathbf{e}_y$ and $+U_0\mathbf{e}_y$, respectively. Origin of the reference frame (x, y) is set on the centerline.

Let $\mathbf{X}_i = \mathbf{X}_i(t)$ be the center of particle i. We denote the fluid domain by

$$\Omega_F(t) = \Omega \Big\backslash \bigcup_{i=1}^{N} \overline{B(\mathbf{X}_i(t), r)}$$

The surrounding fluid is supposed to obey the incompressible Stokes equations in the moving fluid domain $\Omega_F(t)$,

$$\begin{cases} -\mu\triangle\mathbf{u} + \nabla \mathrm{p} = 0 \\ \nabla \cdot \mathbf{u} = 0 \end{cases} \tag{1}$$

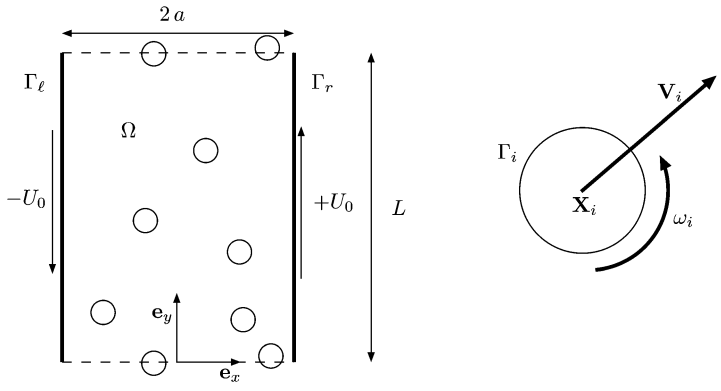

Fig. 1. Notations.

Fig. 1. Notations.