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Modélisation probabiliste d'une expérience ultrasonore : calcul de la dispersion sur les mesures de célérité

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Résumé

Une étude probabiliste de la réflexion d'une onde transitoire à une interface fluide–solide est présentée. La configuration représente une expérience de caractérisation ultrasonore du tissu osseux. Des incertitudes sur les paramètres du solide (os) sont introduites par la méthode paramétrique. Les lois de probabilité pour les variables aléatoires associées sont obtenues par application du principe du maximum d'entropie. Une simulation de Monte Carlo, associée à la méthode de Cagniard–de Hoop pour le calcul de la réponse acoustique, fournit l'estimation d'un paramètre ultrasonore comparable à la vitesse des ondes longitudinales dans le solide. Les résultats mettent en évidence la sensibilité de la densité de probabilité de ce paramètre aux conditions expérimentales. ***Pour citer cet article : K. Macocco et al., C. R. Mecanique 333 (2005).***

Abstract

Probabilistic modelling of an ultrasonic setup: calculation of the dispersion on wave speed measurements. This Note presents a probabilistic model of transient wave reflection at a fluid–solid interface. The configuration represents an ultrasonic experiment used for bone tissue evaluation. The parametric method is used to derive the probabilistic model for the mechanical parameters of the solid (bone); the associated random variables are derived according to the maximum entropy principle. A Monte Carlo simulation, associated with the Cagniard–de Hoop method to calculate the acoustic response, yields the probability density for an output ultrasonic parameter similar to the velocity of longitudinal waves in the solid. Results demonstrate

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the sensitivity of the probability density of this parameter to the experimental setup. ***To cite this article: K. Macocco et al., C. R. Mecanique 333 (2005).***

Mots-clés : Ondes ; Modèle probabiliste paramétrique ; Milieu fluide/solide ; Tissu osseux

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Abridged English version

The axial transmission (AT) technique is used for the evaluation of cortical bone material characteristics [1–3]. The method takes advantage of the lateral wave propagation along the bone-soft tissue interface [4]. This wave which travels at the speed of longitudinal waves of the bone is associated with the first signal monitored (Fig. 1). Experimental signals are very sensitive to material and geometrical parameters of bone [5].

Modelling

As a model of the AT experiment, we investigate a semi-infinite two-media configuration (Fig. 1) which consists of the superposition of a fluid ($x_3 < 0$) and a solid ($x_3 > 0$) along a plane interface. The media are at rest for $t < 0$. At $t = 0$, a cylindrical wave is generated in the fluid along a line source $(S; \mathbf{x}_2)$, where S is a point located at the co-ordinate $x_3 = -h_s$. The configuration is independent of x_2 hence the analysis is conducted in the plane $(O; \mathbf{x}_1, \mathbf{x}_3)$. The acoustic response is calculated at the receivers P_1 and P_2. The line defined by the source and the two receivers makes an angle α with $(O; \mathbf{x}_1)$ and corresponds to the ultrasonic probe orientation. The output parameter of the AT experiment, denoted v, can be thought of as an estimation of the longitudinal wave speed in the solid. This speed is calculated as the ratio of the distance between the receivers and a difference in times of flight as shown in Fig. 2.

The mean problem is defined as follows. The solid is homogeneous, isotropic and linearly elastic. The solid is defined by a mean mass density $\underline{\rho}_s$, a mean Young's modulus $\underline{E}$ and a mean Poisson's ratio $\underline{\nu}$. The fluid is assumed homogeneous and ideal.

The parametric method allows to introduce uncertainties on the parameters. The random variables are $\mathbb{E} = \underline{E}Y_1$, Y_2 and $R = \underline{\rho}_s Y_3$ modelling respectively the Young's modulus, the Poisson's ratio and the mass density of the solid. The mean value of Y_1 and Y_3 is unity. Let $\mathcal{E}\{\cdot\}$ be the mean value. For the present theoretical study, we assume that the following information is given: $\mathbb{E} \in]0, +\infty[$, $\mathcal{E}\{\mathbb{E}\} = \underline{E}$ and $\mathcal{E}\{\frac{1}{\mathbb{E}^2}\} < +\infty$; $Y_2 \in]-1, 1/2[$, $\mathcal{E}\{Y_2\} = \underline{\nu}$, $\mathcal{E}\{\frac{(1-Y_2)^2}{(1+Y_2)^2(1-2Y_2)^2}\} < +\infty$; $R \in]0, +\infty[$, $\mathcal{E}\{R\} = \underline{\rho}_s$ and $\mathcal{E}\{\frac{1}{R^2}\} < +\infty$. Application of the maximum entropy principle [11–13] yields probability densities of Y_i, $(i = 1, \ldots, 3)$, Eqs. (9), (10) and (12).

Method

Only the wave reflected at the fluid–solid interface is considered. The Cagniard–de Hoop [7,8] method yields an explicit analytic expression for the acoustic pressure p at a receiver. The solution of Eq. (8) which can be found in [9,10] is a convolution of the Green's function $\mathcal{G}$ with an input signal.

For each realization of the random vector $\mathbf{Y} = (Y_1, Y_2, Y_3)$ in the Monte Carlo simulation, the corresponding realization of the random variable V modelling v is calculated. Eqs. (14) and (15) give the mean value and the confidence interval of V for the n realizations $V(\theta_i)$.

Results

The fluid has the properties of water and the solid that of bone: $\underline{\rho}_s = 1859\ \mathrm{kg\,m^{-3}}$, $\underline{E} = 20.8 \times 10^9$ and $\underline{\nu} = 0.24$. Distances source-P_1 and $P_1 - P_2$ are 20 mm and 2 mm, respectively and $h_s = 2$ mm. The pressure history at the source is given by Eq. (16) with $f = 1$ MHz. Settings for the parameters of the random model are $\delta_{Y_1} = \delta_{Y_2} =$