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Ergodic dynamics of the stochastic Swift–Hohenberg system ☆

Wei Wang[a], Jianhua Sun[a], Jinqiao Duan[b,*]

[a] *Department of Mathematics, Nanjing University, Nanjing 210008, China*
[b] *Department of Applied Mathematics, Illinois Institute of Technology, Chicago, IL 60616, USA*

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Abstract

The Swift–Hohenberg fluid convection system with both local and nonlocal nonlinearities under the influence of white noise is studied. The objective is to understand the difference in the dynamical behavior in both local and nonlocal cases. It is proved that when sufficiently many of its Fourier modes are forced, the system has a unique invariant measure, or equivalently, the dynamics is ergodic. Moreover, it is found that the number of modes to be stochastically excited for ensuring the ergodicity in the local Swift–Hohenberg system depends *only* on the Rayleigh number (i.e., it does not even depend on the random term itself), while this number for the nonlocal Swift–Hohenberg system relies additionally on the bound of the kernel in the nonlocal interaction (integral) term, and on the random term itself.
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* Corresponding author. Tel.: +1 312 567 5335; fax: +1 312 567 3135.
E-mail addresses: wangshengtaoo@sina.com (W. Wang), jhsun@nju.edu.cn (J. Sun), duan@iit.edu (J. Duan).

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1. Introduction

looseness=-1Density gradient-driven fluid convection arises in geophysical fluid flows in the atmosphere, oceans and the earth's mantle. The Rayleigh–Benard convection is a prototypical model for fluid convection, aiming at predicting spatio-temporal convection patterns. The mathematical model for the Rayleigh–Benard convection involves the Navier–Stokes equations coupled with the transport equation for temperature. When the Rayleigh number is near the onset of the convection, the Rayleigh–Benard convection model may be approximately reduced to an amplitude or order parameter equation, as derived by Swift and Hohenberg [34].

In the literature, most works (e.g., [17,14,28]) on the Swift–Hohenberg model deal with the following evolution equation for order parameter $u(x,t)$, which is localized version of the model originally derived by Swift and Hohenberg [34],

$$u_t = \varrho u - (1+\partial_{xx})^2 u - u^3,$$

where ϱ measures the difference of the Rayleigh number from its critical onset value and the cubic term u^3 is an yet "approximation" of a nonlocal integral term in the original Swift–Hohenberg model [34].

Roberts [32,33] re-examined the rationale for using the Swift–Hohenberg model as a reliable simplified model of the spatial pattern evolution in fluid convection. He argued that, although the localization approximation used in (1) makes some sense, the approximation is deficient in describing some basic features of such systems, and devised via symmetry argument the following modified Swift–Hohenberg equation with nonlocal interactions:

$$u_t = \varrho u - (1+\partial_{xx})^2 u - uG * u^2,$$

where $G * u^2$ is a spatial convolution integral and $G(\cdot)$ is a given radially symmetric positive (or nonnegative) function. We call this function G the kernel for the nonlocal term. In fact, nonlocal integral terms often appear naturally in amplitude equation models for nonequilibrium systems; see, e.g., [25,15,8,23].

Our goal in this paper is to examine the above local and nonlocal Swift–Hohenberg models, by investigating the dynamical difference of both models under random impact as well as under nonlocal interactions.

Fluid systems are often subject to random environmental influences. On the one hand, there is a growing recognition of a role for the inclusion of stochastic effects in the modeling of complex systems. Randomness can have delicate impact on the overall evolution of such systems, for example, noise-induced phase transition or stochastic bifurcation [4], stochastic resonance [19], and noise-induced pattern formation [13,2]. Taking stochastic effects into account is of central importance for the development of mathematical models of such complex phenomena in engineering and science. Macroscopic models in the form of partial differential equations for these systems contain such randomness as stochastic forcing, uncertain parameters, random sources or inputs, and random boundary conditions. Stochastic partial differential equations (SPDEs) are appropriate models for randomly influenced spatially extended systems. On the other hand, the inclusion of such effects has led to interesting new mathematical problems at the interface of probability and partial differential equations.

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