Available online at www.sciencedirect.com
SCIENCE DIRECT®
Nonlinear Analysis Real World Applications
Nonlinear Analysis: Real World Applications 6 (2005) 935 – 954
www.elsevier.com/locate/na

A numerical integrator for a model with a discontinuous sink term: the dynamics of the sexual phase of monogonont rotifera

O. Angulo[a,*], J.C. López-Marcos[b], M.A. López-Marcos[b]

[a] *Departamento de Matemática Aplicada, Escuela Universitaria Politécnica, Universidad de Valladolid, C/Fco. Mendizábal 1, 47014 Valladolid, Spain*

[b] *Departamento de Matemática Aplicada, Facultad de Ciencias, Universidad de Valladolid, C/Fco. Mendizábal 1, 47014 Valladolid, Spain*

Received 1 July 2004; accepted 18 November 2004

Abstract

We present an explicit numerical method especially adapted to approximate the solution of a nonlinear model with a discontinuous sink term: the dynamics of the sexual phase of monogonont rotifera. We show the effectiveness of this numerical technique in the simulation of the dynamics of the solutions. In particular, we show that this method provides a good approximation to the equilibrium solution of the problem. On the other hand, the numerical simulation presented validates the asymptotic behaviour of the numerical solution with regard to the theoretical one.

MSC: 35Q80; 35R05; 65M25; 35B40

Keywords: Age-structured models; Monogonont rotifera; Numerical integration; Equilibrium solution

1. Introduction

Monogonont rotifera are small invertebrate animals which inhabit aquatic media. These species of rotifera have males and females and their reproduction cycle, named *Cyclic*

* Corresponding author. Tel.: +34 983 423000x6805; fax: +34 983 423490.
E-mail addresses: oscar@mat.uva.es (O. Angulo), lopezmar@mac.uva.es (J.C. López-Marcos), malm@mac.uva.es (M.A. López-Marcos).

1468-1218/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.nonrwa.2004.11.007

Parthenogenesis, which is a combination of sexual and asexual reproduction, is of considerable interest and provides a valuable model for the study of sex allocation [11].

This cycle begins after hatching (eclosion) of *resting eggs* (eggs that stay dormant during long periods of time under adverse environmental conditions). These eggs become amictic females (*diploid*: two series of chromosomes). So, in this first asexual phase there is no male presence. There are only amictic females producing diploid eggs that hatch right away to become new amictic females.

The second phase begins induced by environmental factors. There is sexual reproduction and it takes place simultaneously with the other phase. The amictic females begin to produce amictic daughters and mictic (sexual) ones. The virgin mictic daughters produce *haploid eggs* (only one series of chromosomes) which become haploid males after hatching. The males can fertilize the virgin mictic females in the few hours of life of these ones. And, the mated mictic females (the mictic females fertilized) produce resting eggs and the reproductive cycle begins again. More details on this reproduction cycle can be found in the references [10,24].

The model for the sexual phase of monogonont rotifera [12] involves three subclasses in the population: virgin mictic females (male-producing), mated mictic females (resting eggs producing) and haploid males. The population densities $\tilde{v}(\alpha,\tau)$, $m(\alpha,\tau)$ and $\tilde{h}(\alpha,\tau)$, corresponding to the virgin mictic females, the mated mictic females and the haploid males, respectively, depend on the age α and the time τ. The total population of each subclass is, respectively, $\tilde{V}(\tau)=\int_0^\infty \tilde{v}(\alpha,\tau)\,\mathrm{d}\alpha$, $\int_0^\infty m(\alpha,\tau)\,\mathrm{d}\alpha$, and $\tilde{H}(\tau)=\int_0^\infty \tilde{h}(\alpha,\tau)\,\mathrm{d}\alpha$. The population densities satisfy the following system of integro-differential equations:

$$\begin{cases} \tilde{v}_\tau(\alpha,\tau)+\tilde{v}_\alpha(\alpha,\tau)+\tilde{\mu}\,\tilde{v}(\alpha,\tau)=-\tilde{E}\,\tilde{H}(\tau)\,\tilde{v}(\alpha,\tau)\,\chi_{[0,\tilde{T}]}(\alpha), \\ m_\tau(\alpha,\tau)+m_\alpha(\alpha,\tau)+\tilde{\mu}m(\alpha,\tau)=\tilde{E}\,\tilde{H}(\tau)\,\tilde{v}(\alpha,\tau)\,\chi_{[0,\tilde{T}]}(\alpha), \\ \tilde{h}_\tau(\alpha,\tau)+\tilde{h}_\alpha(\alpha,\tau)+\tilde{\delta}\,\tilde{h}(\alpha,\tau)=0, \end{cases} \tag{1}$$

subjected to the boundary conditions

$$\tilde{v}(0,\tau)=B, \quad m(0,\tau)=0, \quad \tilde{h}(0,\tau)=b\int_M^\infty \tilde{v}(\alpha,\tau)\,\mathrm{d}\alpha.$$

The time-independent positive parameters $\tilde{\delta}$ and $\tilde{\mu}$ represent the mortality rate for males and females, respectively, and $\tilde{E}$ is the male–female encounter rate. B is the recruitment rate of mictic females, and b represents the fertility of male-producing mictic females. M denotes the age at maturity for females, and $\tilde{T}$, which satisfies $\tilde{T}\leqslant M$, represents the threshold age of fertilization. Note that $\chi_{[0,\tilde{T}]}(\alpha)$ is the characteristic function of the interval $[0,\tilde{T}]$.

The description of the model is completed with initial conditions for the densities

$$\tilde{v}(\alpha,\tau)=\tilde{v}_0(\alpha), \quad m(\alpha,\tau)=m_0(\alpha), \quad \tilde{h}(\alpha,\tau)=\tilde{h}_0(\alpha).$$

An important remark is that the sink term in (1) exhibits a discontinuity at the age $\alpha=\tilde{T}$.

Note that the evolution of mated mictic females can be considered separately whenever the other two densities were obtained. On the other hand, in order to reduce the number of parameters, the following change of variables is introduced [13]

$$\begin{aligned} &\alpha=Ma, \quad \tau=Mt, \\ &\tilde{v}(\alpha,\tau)=Bv(a,t), \quad \tilde{h}(\alpha,\tau)=BbMh(a,t). \end{aligned}$$