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A two-sex population dynamics model with strong parental care

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Abstract

We present a two-sex age-structured population dynamics deterministic model taking into account parental care of offspring. The model includes a weighted harmonic mean-type pair formation function and neglects the spatial dispersal and separation of pairs. Each sex has pre-reproductive and reproductive age intervals. All adult (of reproductive age) individuals are divided into single males, single females, and permanent pairs. All pairs are of two types: pairs without offspring under parental care at the given time and pairs taking child care. All individuals of pre-reproductive age are divided into young (under parental care) and juvenile (offspring who can live without parental care) groups. It is assumed that births can only occur from couples and after the death of any of the pair partner all offspring under parental care are killed. The model consists of six integro-partial differential equations subject to the conditions of the integral type. A class of separable solutions is studied for this model in the case of time-independent vital rates and a system for macromoments evolving in time is derived in the case of age-independent vital ones.

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1. Introduction

The Sharpe–Lotka–McKendrick–von Foerster (see, e.g., [14]) and Hoppensteadt–Staroverov [5,13] models are well known in mathematical biology. The first one (or its

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Gurtin–MacCamy generalization [3]) is usually used to describe the dynamics of asexual age-structured populations. The other one (or its Hadeler [4] modification involving a maturation period) describes the evolution of populations forming permanent pairs. However, both models do not treat the child care phenomenon which is native for many species of mammals and birds. Most animals care for their offsprings in every possible way. But child care for every species is different. Offspring of mammals and birds spend some time with their mother or both parents, while young offspring of fishes, reptilia, and amphibia are left to one's fate. Mammals and birds feed, warm, and defend their young offspring from enemies [1]. If one of these native parental duties is not realized, young offspring die. For many species of mammals [1], e.g. bear (*Thalarctos maritimus* and *Ursus arctos horribilis*), whale (*Balaenoptera musculus*), and panther (*Panthera onca*), only a female takes care of her young offsprings. For some species of mammals and birds [1], e.g. red fox (*vulpes vulpes*), gnawer (*Dolichotis patagonium*), penguin (*Pygoscelis adeliae*), heron (*Ardea purpurea*), falcon (*Falco columbarius*), and tawny owl (*Strix aluco*), both parents take care of their young offsprings. The female and male of these species of birds hatch in turns. When the female hatches or when she warms her young offsprings (i.e., till when offsprings become fully fledged), her pair partner provides her and his children with food and defends them from enemies in every possible way. In the period of lactation (and in the promiscuous period) the fox-male and gnawer-male provide their female and offspring with food and defend them from enemies. Thus, as a rule, young offsprings of these species die if any of their parents dies.

Moss de Oliveira (see [8] and references therein) was the first to study consequences of child care phenomenon. She introduced parental care into the Penna bit-string model (see, e.g., [8]) which is well known in the computer simulations of biological ageing. Some results of biological ageing obtained by using Monte Carlo simulations for the Penna model are reviewed in [8].

In [10–12] we examined three population dynamics models with child care: two for one-sex population and one for two-sex population forming pairs for the period of mating only.

In the present paper we present a model to describe dynamics of populations which care for their offsprings in couples. To do this, we generalize the Hoppensteadt–Staroverov–Hadeler [5,13,4] model by including parental care of offspring. The model involves a weighted harmonic mean-type pair formation function and neglects the spatial dispersal and separation of pairs. Each sex has pre-reproductive and reproductive age intervals. All adult (of reproductive age) individuals are divided into single males, single females, and permanent pairs. All pairs are of two types: pairs without offspring under parental care at the given time and pairs taking child care. All individuals of pre-reproductive age are divided into the young (under parental care) and juvenile (offspring who can live without parental care) groups. It is assumed that births can only occur from couples and after the death of any of the pair partner all offspring under parental care are killed. It is also assumed that only couples of parents take care for their young offspring. The model consists of six integro-partial differential equations subject to the conditions of an integral type. A class of separable solutions is studied for this model in the Fredrickson form [2] in the case of time-independent vital rates and a system for macromoments evolving in time is derived in the case of age-independent vital ones. This system generalizes the Hadeler equations for macromoments [4]with a maturation period and involves two delays.