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Global existence of solutions to a nonlinear model of sulphation phenomena in calcium carbonate stones

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Abstract

We prove global existence and uniqueness of smooth solutions to a nonlinear system of parabolic equations, which arises to describe the evolution of the chemical aggression due to the action of sulphur dioxide on calcium carbonate stones. This system is not strongly parabolic and only some energy estimates are available. Nevertheless, global (in time) results are proven using a weak continuation principle for the local solutions.

MSC: primary 65M06; secondary 76M20; 76R; 82C40

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1. Introduction

In this paper we investigate the reaction diffusion system

$$\begin{cases} \partial_t(\varphi(c)s) = \partial_x(\varphi(c)\partial_x s) - \varphi(c)cs, \\ \partial_t c = -\varphi(c)cs, \end{cases} \tag{1.1}$$

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for $(x,t) \in \mathbb{R} \times [0,T]$ $(T>0)$, under the initial conditions

$$s(x,0) = s_0(x), \quad c(x,0) = c_0(x). \tag{1.2}$$

This model has been introduced in [2] to describe the transformation in time of $CaCO_3$ (calcium carbonate) stones under the chemical aggression due to the SO_2 (sulphur dioxide). In (1.1), s stands for the porous concentration of SO_2, namely the concentration taken with respect to the volume of the pores, and c for the local density of $CaCO_3$.

In the following, we always assume that the initial data $s_0,\ c_0$ are nonnegative functions such that

$$0 \leqslant s_0(x) \leqslant S_0, \quad 0 < m \leqslant c_0(x) \leqslant C_0, \quad s_0 \in L^2(\mathbb{R}), \quad C_0 - c_0 \in H^1(\mathbb{R}). \tag{1.3}$$

The function φ (the porosity) is a linear function of the density c, $\varphi(c) = A + Bc$, which is strictly positive on the interval $[0, C_0]$; actually there are two strictly positive constants $\varphi_m < \varphi_M$, such that

$$\varphi_m \leqslant \varphi(c) \leqslant \varphi_M, \quad \text{for all } c \in [0, C_0].$$

In particular, this implies that $\min\{A, A + BC_0\} \geqslant \varphi_m > 0$.

There is an extensive chemical literature about the deterioration mechanisms of natural building stones [6,9,10,14,15], which deals with problems concerning modern and historical buildings. Sulphur dioxide and nitrogen oxides emitted into the atmosphere by sources related to industry, transportation and heating, react with calcium carbonate stones to form sulphates and nitrates, which, due to their solubility in water, may be drained away or, if protected from the rain, may form crusts, that eventually exfoliate, see [6]. Standard methods developed for studying the evolution of this kind of damage use the statistic determination on the ratio dose/response of the materials.

Model (1.1), proposed in [2], is based on the different framework of hydrodynamical models, and gives the possibility of an accurate numerical approximation of the equations by finite elements or finite differences methods. Other advantages of this formulation are: a better understanding of the involved physical processes, the possibility to adapt the model to more complex situations, where other damage factors are involved, and an affective calibration against experimental data, see [7] for some preliminary results. One main factor is the possibility of a determination of the time asymptotic regime in one space dimension, which has been numerically explored in [2].

In this paper we are interested in the proof of existence, uniqueness and regularity of global solutions to problem (1.1)–(1.2). In our investigation we have to take into account that we are dealing with a system of nonlinear parabolic equations, which is not parabolic in the sense of Petrovskii [18]. Nevertheless, existence and uniqueness of local solutions are known, at least for smooth initial data, see for instance [5].

To obtain global solutions, we have to use some kind of continuation principle. Standard criteria, as the uniform boundedness of some Hölder-norm of the solutions [18], or the dependence of the life-span only on the L^∞-norm of the data [16], do not work in our case. Besides, system (1.1) does not verify the coupling conditions of Kawashima–Shizuta [13] for the global existence of solutions to general systems of hyperbolic–parabolic type.