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On global stability of the scalar Chaboche models

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Abstract

We present global stability conditions for systems of differential equations, which arise as models of multisurface stress–strain laws of the so-called nonlinear kinematic hardening type and include the scalar stop operator. In addition, we analyze some properties of the models, in particular, monotonicity and contracting properties, and consider periodic solutions in case of periodic inputs.

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1. Introduction

We consider systems referred to as the scalar Chaboche models of the stress–strain laws in elastoplasticity, also characterized as multisurface rate independent models of the nonlinear kinematic hardening type. Mechanical aspects and foundations of these models, which in

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particular take into account the so-called ratchetting effect, can be found, e.g., in [6–9,13]. We use the formulation of the model in the form developed in [2–4] and based on the approach of the operator theory of hysteresis nonlinearities [5,10,11,14]. Here, in the main case of the finite number of yield surfaces, the model becomes a system of ordinary differential equations coupled with the input–output relation prescribed by the stop hysteresis operator. We consider systems with scalar-valued inputs and outputs (interpreted as time-dependent stress and strain); this corresponds to the uniaxial plasticity.

As it was proved in [4], the Chaboche models are well-posed. Particularly, it means that for a given input each initial state defines a unique solution of the system on the positive semiaxis and solutions depend continuously on the initial states (x_0, σ_0); here $x_0 \in [-r, r]$ is the initial state of the stop; the other initial value $\sigma_0 = \sigma_0(\cdot)$ is an integrable function defined on some measure space (in the discrete case, σ_0 is a vector). The main problem of this paper is the analysis of the global stability. We show that if some function defined by the input θ of the system has the unbounded variation on the positive semiaxis, then after some moment t' the x-components of all solutions equal each other and the distance between σ-components for every pair of solutions decreases and vanishes at infinity.

In case of a periodic input θ, our condition for the global stability becomes necessary and sufficient and takes the simple form

$$|\theta(t') - \theta(t'')| > 2r \quad \text{for some } \; t' > t'' \geqslant 0. \tag{1}$$

This estimate guarantees that the distance between σ-components for every pair of solutions decreases exponentially. If some periodic input satisfies the opposite estimate $|\theta(t') - \theta(t'') \leqslant 2r$ for all t', t'', then the system has a continuum of trivial periodic solutions and consequently is not globally stable.

An important point of our method is the reduction of the model to a simpler system. In the equation for the x-component, we replace the input u of the stop (the function u enters also the other equations and is unknown a priori) by the given input θ of the model and show that this leads to the equivalent system. As a result, we conclude that the x-component of any solution is prescribed by the initial value x_0 of this component and the input θ of the model. This accounts also for the fact that x-components of all solutions equal each other after the moment t' if relation (1) is valid.

We do not consider vector Chaboche models. They are described by systems of the same form, but with vector-valued functions and with the vector stop operator in place of scalar ones. It seems that the method developed in this paper can hardly be adjusted to the vector case; in particular, our reduction to a simpler system does not work. The reason is first of all that we use specific properties of the scalar stop operator, which either do not have analogs or differ from that of the stop of any dimension greater than one. We do not know reasonably general conditions for the global stability of vector models. Simple examples show that already in two-dimensional case the Armstrong–Frederick model (the simplest Chaboche model with one yield surface) with a periodic input satisfying (1) can be not globally stable.

The paper is organized as follows. In the next section, we give the definitions of the stop operator and the Chaboche model and observe some of their simple properties. Section 3 contains the main results on the global stability. We consider separately periodic and nonperiodic inputs (the existence of at least one periodic solution in case of periodic inputs,