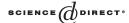


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A lower semicontinuity result for some integral functionals in the space *SBD*

François Ebobisse*

Department of Maths and Applied Maths, University of Cape Town, Rondebosch 7701, South Africa

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Abstract

The purpose of this paper is to study the lower semicontinuity with respect to the strong L^1 -convergence, of some integral functionals defined in the space SBD of special functions with bounded deformation. Precisely, we prove that, if $u \in SBD(\Omega)$, $(u_h) \subset SBD(\Omega)$ converges to u strongly in $L^1(\Omega, \mathbb{R}^n)$ and the measures $|E^ju_h|$ converge weakly * to a measure v singular with respect to the Lebesgue measure, then

$$\int_{\Omega} f(x, \mathscr{E}u) \, \mathrm{d}x \leqslant \liminf_{h \to \infty} \int_{\Omega} f(x, \mathscr{E}u_h) \, \mathrm{d}x$$

provided the integrand f satisfies a weak convexity property and standard growth assumptions of order n > 1.

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^{*} Tel.: +27 21 650 5734; fax: +27 21 650 2334. E-mail address: ebobisse@maths.uct.ac.za.

1. Introduction

Our goal in this paper is to extend in the framework of functions with bounded deformation, the following theorem by Ambrosio [2] for integral functionals defined in the space *SBV* of special functions of bounded variation.

Theorem 1.1. Let $\Omega \subset \mathbb{R}^n$ be an open set and let $f: \Omega \times \mathbb{R}^k \times \mathbb{R}^{n \times k}$ be a Carathéodory function satisfying:

(i) for a.e. every $x \in \Omega$, for every $(u, \xi) \in \mathbb{R}^k \times \mathbb{R}^{n \times k}$,

$$|\xi|^p \leqslant f(x, u, \xi) \leqslant a(x) + \Psi(|u|)(1 + |\xi|^p),$$

where p > 1, $a \in L^1(\Omega)$ and the function $\Psi: [0, \infty) \to [0, \infty)$ is continuous;

(ii) for a.e. every $x \in \Omega$ and every $u \in \mathbb{R}^k$, $f(x, u, \cdot)$ is quasi-convex.

Then for every $u \in SBV(\Omega, \mathbb{R}^k)$ and any sequence $(u_h) \subset SBV(\Omega, \mathbb{R}^k)$ converging to u in $L^1_{loc}(\Omega, \mathbb{R}^k)$ and such that

$$\sup_{h} \mathcal{H}^{n-1}(S_{u_h}) < \infty \tag{1.1}$$

we have

$$\int_{\Omega} f(x, u, \nabla u) dx \leqslant \liminf_{h \to \infty} \int_{\Omega} f(x, u_h, \nabla u_h) dx.$$

Theorem 1.1 extends in the *SBV* setting a classical lower semicontinuity result by Acerbi and Fusco [1] in the Sobolev space $W^{1,p}(\Omega)$.

Later Kristensen in [19] extended Theorem 1.1 under the weaker assumptions

$$\sup_{h} \int_{S_{u_h}} \theta(|u_h^+ - u_h^-|) \, \mathrm{d}\mathcal{H}^{n-1} < \infty \tag{1.2}$$

for some function θ such that $\theta(r)/r \to \infty$ as $r \to 0^+$, and f is a normal integrand, i.e. for a.e. $x \in \Omega$, $f(x, \cdot, \cdot)$ is lower semicontinuous in $\mathbb{R}^k \times \mathbb{R}^{n \times k}$ and there exists a Borel function $\tilde{f}: \Omega \times \mathbb{R}^k \times \mathbb{R}^{n \times k} \to [0, \infty]$ such that $f(x, \cdot, \cdot) = \tilde{f}(x, \cdot, \cdot)$.

In the proof of Theorem 1.1 as well as in the Acerbi–Fusco result, the use of Lusintype approximation of functions in the given space (BV or Sobolev spaces) by Lipschitz continuous functions is crucial.

Recently, Theorem 1.1 has been extended by Fonseca et al. [16] to functionals depending also on the Hessian matrices.

In this paper we deal with first-order variational problem, but with integral functionals depending explicitly on the symmetrized derivative $Eu := (Du + Du^{T})/2$ and defined in the space SBD of special functions with bounded deformation.

The main result of the paper is the following lower semicontinuity theorem.

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