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A lower semicontinuity result for some integral functionals in the space *SBD*

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Abstract

The purpose of this paper is to study the lower semicontinuity with respect to the strong L^1-convergence, of some integral functionals defined in the space *SBD* of special functions with bounded deformation. Precisely, we prove that, if $u \in SBD(\Omega)$, $(u_h) \subset SBD(\Omega)$ converges to u strongly in $L^1(\Omega, \mathbb{R}^n)$ and the measures $|E^j u_h|$ converge weakly $*$ to a measure ν singular with respect to the Lebesgue measure, then

$$\int_\Omega f(x, \mathscr{E}u)\,\mathrm{d}x \leqslant \liminf_{h\to\infty} \int_\Omega f(x, \mathscr{E}u_h)\,\mathrm{d}x$$

provided the integrand *f* satisfies a weak convexity property and standard growth assumptions of order $p > 1$.

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1. Introduction

Our goal in this paper is to extend in the framework of functions with bounded deformation, the following theorem by Ambrosio [2] for integral functionals defined in the space *SBV* of special functions of bounded variation.

Theorem 1.1. *Let $\Omega \subset \mathbb{R}^n$ be an open set and let $f : \Omega \times \mathbb{R}^k \times \mathbb{R}^{n\times k}$ be a Carathéodory function satisfying*:

(i) *for a.e. every $x \in \Omega$, for every $(u, \xi) \in \mathbb{R}^k \times \mathbb{R}^{n\times k}$,*

$$|\xi|^p \leqslant f(x, u, \xi) \leqslant a(x) + \Psi(|u|)(1 + |\xi|^p),$$

where $p > 1$, $a \in L^1(\Omega)$ and the function $\Psi : [0, \infty) \to [0, \infty)$ is continuous;

(ii) *for a.e. every $x \in \Omega$ and every $u \in \mathbb{R}^k$, $f(x, u, \cdot)$ is quasi-convex.*

Then for every $u \in SBV(\Omega, \mathbb{R}^k)$ and any sequence $(u_h) \subset SBV(\Omega, \mathbb{R}^k)$ converging to u in $L^1_{\text{loc}}(\Omega, \mathbb{R}^k)$ and such that

$$\sup_h \mathscr{H}^{n-1}(S_{u_h}) < \infty \tag{1.1}$$

we have

$$\int_\Omega f(x, u, \nabla u)\, \mathrm{d}x \leqslant \liminf_{h\to\infty} \int_\Omega f(x, u_h, \nabla u_h)\, \mathrm{d}x.$$

Theorem 1.1 extends in the *SBV* setting a classical lower semicontinuity result by Acerbi and Fusco [1] in the Sobolev space $W^{1,p}(\Omega)$.

Later Kristensen in [19] extended Theorem 1.1 under the weaker assumptions

$$\sup_h \int_{S_{u_h}} \theta(|u_h^+ - u_h^-|)\, \mathrm{d}\mathscr{H}^{n-1} < \infty \tag{1.2}$$

for some function θ such that $\theta(r)/r \to \infty$ as $r \to 0^+$, and *f* is a normal integrand, i.e. for a.e. $x \in \Omega$, $f(x, \cdot, \cdot)$ is lower semicontinuous in $\mathbb{R}^k \times \mathbb{R}^{n\times k}$ and there exists a Borel function $\tilde{f} : \Omega \times \mathbb{R}^k \times \mathbb{R}^{n\times k} \to [0, \infty]$ such that $f(x, \cdot, \cdot) = \tilde{f}(x, \cdot, \cdot)$.

In the proof of Theorem 1.1 as well as in the Acerbi–Fusco result, the use of Lusin-type approximation of functions in the given space (*BV* or Sobolev spaces) by Lipschitz continuous functions is crucial.

Recently, Theorem 1.1 has been extended by Fonseca et al. [16] to functionals depending also on the Hessian matrices.

In this paper we deal with first-order variational problem, but with integral functionals depending explicitly on the symmetrized derivative $Eu := (Du + Du^{\mathrm{T}})/2$ and defined in the space *SBD* of special functions with bounded deformation.

The main result of the paper is the following lower semicontinuity theorem.