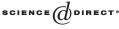


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Convergence of a Lagrangian scheme for a compressible Naviers–Stokes model defined on a domain depending on time

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Abstract

This paper deals with the numerical resolution of the Navier–Stokes equations defined on a time dependent domain. We give an existence result for a fluid-structure interaction problem in which the boundary is governed by a thin plate operator. We propose to solve the fluid equations with the characteristics method. We approach the total derivative with a "regularized" finite difference scheme and we study the convergence of the discrete problem towards the continuous one. © 2005 Elsevier Ltd. All rights reserved.

Keywords: Compressible Navier-Stokes; Lagrangian scheme

1. Introduction

This paper deals with a fluid-structure interaction problem in which the structure surrounds a compressible viscous fluid. We consider the case of the compressible isentropic Navier–Stokes equations ([13], p. 236 with advection term) coupled with a thin plate operator. Even in the case of weak deformations we need to take into account domain variations in order to obtain mass conservation and energy-type estimates [9,8]. As a consequence, we must solve the fluid equations on a non cylindrical domain. In [9,8] the authors propose a method based on a cylindrical regularization of the momentum equation and show that the problem admits a solution for an adiabatic coefficient $\gamma = 1$ and for small data. In this

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paper we propose a Lagrangian scheme to solve the bidimensional fluid equations (\mathcal{F}) in the case $\gamma > 1$ and for small data. Euler scheme is not appropriate for the discretization of this kind of problem since we work on a non cylindrical domain. Moreover the Lagrangian description allows to follow each particle in its motion and thus to take naturally into account the boundary variations. In [3], Bercovier et al. present several numerical tests on parabolic-hyperbolic problems to evaluate some implementation procedures based on "finite elements-characteristics" schemes. One can also refer to the "characteristics-Galerkin" method proposed by Pironneau and Huberson [18] and to the "transport-diffusion algorithm" proposed by Pironneau [17] to solve the Navier–Stokes equations. Let us mention on the subject the work of Süli who studies in [20] the finite difference approximation of the Lagrangian material derivative along trajectories with the Galerkin finite element method. More recently, we can cite the paper of Achdou and Guermond [1] on the convergence analvsis of the Lagrange-Galerkin scheme for the incompressible Navier-Stokes equations. All these works concern problems defined on domains independent of time. Generally, the Arbitrary Lagrangian–Eulerian method is preferred to the methods coupling the characteristics with a space discretization as Galerkin or finite elements. In particular the mesh obtained after few iteration can be bad. Numerous papers propose to solve Navier–Stokes equations in a moving domain by using the Arbitrary Lagrangian Eulerian method. We can cite for instance Donéa et al. who give a survey of this method [5]. Concerning the fluid-structure interaction, we refer to Donéa et al. [6], Grandmont and Maday [10] and Quarteroni et al. [19].

Our purpose is to build approximate solutions of a penalized problem $(\mathscr{F}^{\varepsilon}) (\mathscr{F}^{\varepsilon} \xrightarrow{\varepsilon \to 0} \mathscr{F})$. To approach the total derivative, we propose a finite difference approximation at which we add a regularizing operator *A* depending on the discretization step and vanishing as this step goes to 0⁺. The Lagrangian description is physically well adapted to describe the boundary motion. The operator *A* gives the necessary compactness to justify all the calculations and to pass to the limit inside the equations. Moreover, this operator gives a meaning to the discretization since it allows to show that a particle do not leave the domain from a time step to another. The Lagrangian discretization allows us to circumvent the difficulties linked with the nonlinear term (advection) in the momentum equation and to lead us to solve a "nice" semi-linear stationary problem.

We set $Q_p = \Omega_p \times [0, T[$ where Ω_p is an open subset of \mathbb{R} which physically represents the plate at rest, $\Sigma_1 = \Gamma_1 \times [0, T[$, where Γ_1 is a part of fluid boundary assumed to be fixed and smooth enough (with meas $(\Gamma_1) \neq 0$), $\Sigma_2 = \bigcup_{t \in [0, T[} \Gamma_2(t) \times \{t\}$, where $\Gamma_2(t)$ is the plate deformation at time t. $\Gamma_2(t) = \Omega_p$ when there is no deformation. We set $Q = \bigcup_{t \in [0, T[} \Omega_t \times \{t\}$ the three-dimensional domain where $\Omega_t = \bigcup_{x \in \Omega_p}]d(x, t)$, 1[is the domain occupied by the fluid at times t and d(x, t) the plate motion. The section $s \to \Omega_s = Q \cap \{t = s\}$ is continuous and Ω_t is never empty if d is a continuous function of x and t (Fig. 1).

The fluid is governed by the following compressible isentropic Navier–Stokes problem [13] with velocity $u = (u_x, u_z) \in \mathbb{R}^2$ and density $\rho \in \mathbb{R}^+$

$$(\mathscr{F}) \begin{cases} \frac{\partial u}{\partial t} + (u \cdot \nabla)u - \mu \Delta u - \lambda \nabla \operatorname{div} u + a \nabla \rho^{\gamma} = 0, & \text{in } Q\\ \frac{\partial \rho}{\partial t} + \operatorname{div} (\rho u) = 0, & \text{in } Q, \end{cases}$$

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