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Convergence of a Lagrangian scheme for a compressible Naviers–Stokes model defined on a domain depending on time

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Abstract

This paper deals with the numerical resolution of the Navier–Stokes equations defined on a time dependent domain. We give an existence result for a fluid-structure interaction problem in which the boundary is governed by a thin plate operator. We propose to solve the fluid equations with the characteristics method. We approach the total derivative with a “regularized” finite difference scheme and we study the convergence of the discrete problem towards the continuous one.

Keywords: Compressible Navier–Stokes; Lagrangian scheme

1. Introduction

This paper deals with a fluid-structure interaction problem in which the structure surrounds a compressible viscous fluid. We consider the case of the compressible isentropic Navier–Stokes equations ([13], p. 236 with advection term) coupled with a thin plate operator. Even in the case of weak deformations we need to take into account domain variations in order to obtain mass conservation and energy-type estimates [9,8]. As a consequence, we must solve the fluid equations on a non cylindrical domain. In [9,8] the authors propose a method based on a cylindrical regularization of the momentum equation and show that the problem admits a solution for an adiabatic coefficient $\gamma = 1$ and for small data. In this

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paper we propose a Lagrangian scheme to solve the bidimensional fluid equations $(\mathscr{F})$ in the case $\gamma > 1$ and for small data. Euler scheme is not appropriate for the discretization of this kind of problem since we work on a non cylindrical domain. Moreover the Lagrangian description allows to follow each particle in its motion and thus to take naturally into account the boundary variations. In [3], Bercovier et al. present several numerical tests on parabolic-hyperbolic problems to evaluate some implementation procedures based on “finite elements-characteristics” schemes. One can also refer to the “characteristics-Galerkin” method proposed by Pironneau and Huberson [18] and to the “transport-diffusion algorithm” proposed by Pironneau [17] to solve the Navier–Stokes equations. Let us mention on the subject the work of Süli who studies in [20] the finite difference approximation of the Lagrangian material derivative along trajectories with the Galerkin finite element method. More recently, we can cite the paper of Achdou and Guermond [1] on the convergence analysis of the Lagrange–Galerkin scheme for the incompressible Navier–Stokes equations. All these works concern problems defined on domains independent of time. Generally, the Arbitrary Lagrangian–Eulerian method is preferred to the methods coupling the characteristics with a space discretization as Galerkin or finite elements. In particular the mesh obtained after few iteration can be bad. Numerous papers propose to solve Navier–Stokes equations in a moving domain by using the Arbitrary Lagrangian Eulerian method. We can cite for instance Donéa et al. who give a survey of this method [5]. Concerning the fluid-structure interaction, we refer to Donéa et al. [6], Grandmont and Maday [10] and Quarteroni et al. [19].

Our purpose is to build approximate solutions of a penalized problem $(\mathscr{F}^{\varepsilon})$ $(\mathscr{F}^{\varepsilon} \xrightarrow{\varepsilon\to 0} \mathscr{F})$. To approach the total derivative, we propose a finite difference approximation at which we add a regularizing operator A depending on the discretization step and vanishing as this step goes to 0^+. The Lagrangian description is physically well adapted to describe the boundary motion. The operator A gives the necessary compactness to justify all the calculations and to pass to the limit inside the equations. Moreover, this operator gives a meaning to the discretization since it allows to show that a particle do not leave the domain from a time step to another. The Lagrangian discretization allows us to circumvent the difficulties linked with the nonlinear term (advection) in the momentum equation and to lead us to solve a “nice” semi-linear stationary problem.

We set $Q_{\rm p} = \Omega_{\rm p} \times]0, T[$ where $\Omega_{\rm p}$ is an open subset of $\mathbb{R}$ which physically represents the plate at rest, $\Sigma_1 = \Gamma_1 \times]0, T[$, where Γ_1 is a part of fluid boundary assumed to be fixed and smooth enough (with meas $(\Gamma_1) \neq 0$), $\Sigma_2 = \cup_{t\in]0,T[}\Gamma_2(t) \times \{t\}$, where $\Gamma_2(t)$ is the plate deformation at time t. $\Gamma_2(t) = \Omega_{\rm p}$ when there is no deformation. We set $Q = \cup_{t\in]0,T[}\Omega_t \times \{t\}$ the three-dimensional domain where $\Omega_t = \cup_{x\in\Omega_{\rm p}}]d(x,t), 1[$ is the domain occupied by the fluid at times t and $d(x,t)$ the plate motion. The section $s \to \Omega_s = Q \cap \{t = s\}$ is continuous and Ω_t is never empty if d is a continuous function of x and t (Fig. 1).

The fluid is governed by the following compressible isentropic Navier–Stokes problem [13] with velocity $u = (u_x, u_z) \in \mathbb{R}^2$ and density $\rho \in \mathbb{R}^+$

$$(\mathscr{F}) \begin{cases} \dfrac{\partial u}{\partial t} + (u \cdot \nabla)u - \mu\Delta u - \lambda\nabla \operatorname{div} u + a\nabla\rho^{\gamma} = 0, & \text{in } Q \\ \dfrac{\partial \rho}{\partial t} + \operatorname{div}(\rho u) = 0, & \text{in } Q, \end{cases}$$