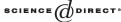


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Blow-up solutions of inhomogeneous nonlinear Schrödinger equations on torus ☆

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Abstract

In this paper, we study blow-up solutions to the Cauchy problem of the inhomogeneous nonlinear Schrödinger equation

$$\partial_t u = i(f(x)\Delta u + \nabla f(x) \cdot \nabla u + k(x)|u|^2 u)$$

on \mathbb{T}^2 . We present the L^2 -concentration property for general initial data and investigate the L^2 -minimality.

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1. Introduction

Recently, Schrödinger-type equations with variable coefficients have been of considerable interest among both mathematicians and physicists, and some remarkable progress on the Cauchy problem has been made, see [5–7,11,12,14,15,22,24,28] and references therein for well-posedness, [18,23] for blow-up analysis, and [25] for Strichartz-type inequalities. In this paper we consider the following *inhomogeneous* cubic nonlinear Schrödinger equation (NLS) on the flat torus $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ (i.e., the space-periodic case):

$$\partial_t u = i(f(x)\Delta u + \nabla f(x) \cdot \nabla u + k(x)|u|^2 u), \tag{1.1}$$

together with the initial condition

$$u(0,\cdot) = u_0(\cdot),\tag{1.2}$$

where f(x) and k(x) are (non-constant) real-valued functions on \mathbb{T}^2 . It is easy to see that the cubic nonlinearity is critical for blow-up in space-dimension two for focusing inhomogeneous NLS (while $|u|^{4/N}u$ for space-dimension N).

When the functions $f \equiv k \equiv 1$, the Cauchy problem (1.1)–(1.2) is the normal (homogeneous) critical NLS with space dimension two, which has been extensively studied by many mathematicians. The reader is referred to Refs. [2,9,27] for a comprehensive review for both non- and space-periodic cases, [3,8,13] for the bounded domains case (with Dirichlet boundary condition), and [19,20] for recent stunning progress on blow-up analysis. Antonini [1] studied the L^2 -minimal blow-up solutions on \mathbb{T}^N . He showed the corresponding L^2 -concentration property and gave a sharp lower bound of blow-up rate. His argument is based on the analysis on the local virial identity and the conservation of mass, energy and momentum.

For general real-valued functions f(x) and k(x), as in the homogeneous case, one can easily check that solutions of (1.1)–(1.2) obey conservation of mass and energy as follows:

$$\int_{\mathbb{T}^2} |u(t,x)|^2 dx = \int_{\mathbb{T}^2} |u_0(x)|^2 dx,$$
(1.3)

$$E_{f,k}(u(t)) = E_{f,k}(u_0),$$
 (1.4)

where

$$E_{f,k}(u) = \frac{1}{2} \int_{\mathbb{T}^2} f(x) |\nabla u(x)|^2 dx - \frac{1}{4} \int_{\mathbb{T}^2} k(x) |u(x)|^4 dx.$$

Momentum is no longer conserved in this case, however.

We first recall the following existence and uniqueness result for the problem (1.1)–(1.2):

Theorem 0 (Pang et al. [22]). Let $s_0 \ge 4$ be an integer. Suppose $f \in C^{s_0+1}(\mathbb{T}^2)$ and $k \in C^{s_0}(\mathbb{T}^2)$ are real functions and f(x) has no zero points. Then, given $u_0 \in H^{s_0}(\mathbb{T}^2)$, the Cauchy problem (1.1)–(1.2) admits a unique local smooth solution $u \in L^{\infty}([0, T), H^{s_0}(\mathbb{T}^2))$. Moreover, the solution is global in the sense that $u \in L^{\infty}_{loc}([0, \infty), H^{s_0}(\mathbb{T}^2))$ provided $f \cdot k \le 0$ or $\|u_0\|_{L^2}$ is small enough.

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