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Periodic solutions for p-Laplacian differential equation with multiple deviating arguments

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Abstract

By employing Mawhin's continuation theorem, the existence of periodic solutions of the p-Laplacian differential equation with multiple deviating arguments

$$(\varphi_p(x'(t)))' + f(x(t))x'(t) + \sum_{j=1}^{n} \beta_j(t)g(x(t-\gamma_j(t))) = e(t)$$

under various assumptions are obtained.

Keywords: Periodic solution; Mawhin's continuation theorem; Deviating argument

1. Introduction

In recent years, there have been a number of results on the existence of periodic solutions for delay differential equations. For example, in [1,8–11,13], the following types of

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second-order scalar differential equations with delay:

$$x''(t) + g(x(t-\tau)) = p(t), \tag{1.1}$$

$$x''(t) + m^2 x(t) + g(x(t-\tau)) = p(t), \tag{1.2}$$

$$x''(t) + f(x(t))x'(t) + g(x(t-\tau(t))) = p(t), \tag{1.3}$$

$$x''(t) + f(t, x(t), x(t-\tau_0(t)))x'(t) + \beta(t)g(x(t-\tau_1(t))) = p(t), \tag{1.4}$$

and

$$x''(t) + f(x(t))x'(t) + \sum_{j=1}^{n} \beta_j(t)g(x(t-\gamma_j(t))) = p(t) \tag{1.5}$$

have been studied. The main technique used in these works is to convert the problem into the abstract form $Lx = Nx$, with L being a non-invertible linear operator. Here, the crux is that the leading terms of these equations, that is, the one-dimensional Laplacian $x''(t)$, are linear in the unknown function x so that Mawhin's continuation theorem [6] applies and existence of solutions follows.

Now as the p-Laplacian of a function comes frequently into play in many practical situations (for example, it is used to describe fluid mechanical and nonlinear elastic mechanical phenomena), it is natural to try and consider the existence of solutions of p-Laplacian equations, that is, differential equations with the leading term being a p-Laplacian $(\varphi_p(x'(t)))'$, where $\varphi_p(u) = |u|^{p-2}u$. Since $x''(t) = \varphi_2(x'(t))'$, p-Laplacians cover the usual Laplacian as a special case. So it should be interesting to consider the aforesaid equations with $x''(t)$ being replaced by $(\varphi_p(x'(t)))'$ and in fact, there have already been a few results for p-Laplacian equations, for example, see [4,14] and references cited therein. But for the existence of solutions of p-Laplacian boundary value problems at resonance or p-Laplacian differential equations with delay (or deviating argument), as far as we are aware of, there have been little results until the very recent works in [2,3,7]. The major difficulty in this direction is that except for $p = 2$, $(\varphi_p(x'(t)))'$ is no longer linear and so the usual technique of using Mawhin's continuation theorem does not apply directly. In order to get around with this difficulty, Ge and Ren [7] obtained an extension of Mawhin's continuation theorem and applied it to boundary value problems with a p-Laplacian. At the same time, Cheung and Ren [2,3] designed a new technique of tackling the problem, namely, to translate the p-Laplacian equation into a two-dimensional system for which Mawhin's continuation theorem can be applied.

On the other hand, as multi-delays exist naturally in most non-simple situations, such phenomena are worth investigating. Recent results in this direction include [4,5,10,12,15]. In this paper, following the line of Cheung-Ren in [2,3] we consider the p-Laplacian differential equation with multiple deviating arguments

$$(\varphi_p(x'(t)))' + f(x(t))x'(t) + \sum_{j=1}^{n} \beta_j(t)g(x(t-\gamma_j(t))) = e(t), \tag{1.6}$$

where $p > 1$ is a constant; $\varphi_p : \mathbb{R} \to \mathbb{R}$, $\varphi_p(u) = |u|^{p-2}u$ is a one-dimensional p-Laplacian; $f, g, e, \beta_j \in C(\mathbb{R}, \mathbb{R})$, $j = 1, 2, \ldots, n$, are periodic with period $T > 0$,