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Solutions of two-point BVPs at resonance for higher order impulsive differential equations☆

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Abstract

In this paper, the resonance two-point boundary value problems for impulsive 2n-order differential equation

$$\begin{cases} x^{(2n)}(t) = f(t, x(t), x'(t), \ldots, x^{(2n-1)}(t)) & 0 < t < 1 \\ \Delta x^{(i)}(s_k) = I_{i,k}(x(s_k^-), \ldots, x^{(2n-1)}(s_k^-)), & i = 1, \ldots, 2n-1,\ k = 1, \ldots, p, \end{cases}$$

with following two-point boundary value conditions

$$x^{(2i+1)}(0) = x^{(2i+1)}(1) = 0, \quad i = 0, \ldots, n-1,$$

and for n-order differential equation

$$\begin{cases} x^{(n)}(t) = f(t, x(t), x'(t), \ldots, x^{(n-1)}(t)) & 0 < t < 1 \\ \Delta x^{(i)}(s_k) = I_{i,k}(x(s_k^-), \ldots, x^{(2n-1)}(s_k^-)), & i = 1, \ldots, n-1,\ k = 1, \ldots, p, \end{cases}$$

with following periodic boundary value conditions

$$x^{(i)}(0) = x^{(i)}(1), \quad i = 0, \ldots, n-1$$

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are considered. Sufficient conditions which guarantee the existence of at least one solution for these problems are established. The interest is that we allow the degree of variables of f to be greater than 1. The methods used and results obtained are new and they shows us that the solvability of these two problems are very much alike.

MSC: 34B15

Keywords: Resonance; Two-point boundary value problem; Higher-order differential equation; Solution; Fixed point theorem

1. Introduction

Let $f:[0,1]\times R^4$ be continuous and $e\in L^1[0,1]$. In [24], the authors considered the solvability of two-point boundary value problem for fourth-order differential equation

$$x^{(4)}(t)=f(t,x(t),x'(t),x''(t),x'''(t))+e(t),\quad t\in(0,1) \tag{1}$$

subject to following boundary value conditions

$$x'(0)=x'(1)=x'''(0)=x'''(1)=0. \tag{2}$$

The boundary value problem of this form was used to understand the static equilibrium of elastic beam supported by sliding clamps. We refer the reader to [13,14] and the references therein.

Gupta, in [14], studied the solvability of the following boundary value problem

$$\begin{cases} -y^{(4)}+g(t,y(t))=e(t),\quad t\in(0,1),\\ y'(0)=y'(\pi)=y'''(0)=y'''(\pi)=0, \end{cases} \tag{3}$$

On the other hand, there are many papers concerning the existence of solutions or positive solutions of 2n-order differential equations or n-order differential equations subjected to different kind of boundary value conditions, see, [1,2,4,8,9,23,28–30] and the references therein. The investigation of solvability of two-point or multi-point boundary value problems for differential equations is of interest.

Unlike Lidstone boundary value problems for 2n-order differential equations,

$$\begin{cases} x^{(2n)}(t)=f(t,x(t),x'(t),\ldots,x^{(2n-1)}(t))\quad 0<t<1,\\ x^{(2i)}(0)=x^{(2i)}(1)=0,\quad i=1,\ldots,n-1, \end{cases} \tag{4}$$

and

$$\begin{cases} x^{(2n)}(t)=f(t,x(t),x'(t),\ldots,x^{(2n-1)}(t))\quad 0<t<1,\\ x^{(2i+1)}(1)=x^{(2i)}(0)=0,\quad i=1,\ldots,n-1, \end{cases} \tag{5}$$

which were studied by many authors, the following boundary value problems

$$\begin{cases} x^{(2n)}(t)=f(t,x(t),x'(t),\ldots,x^{(2n-1)}(t))\quad 0<t<1,\\ x^{(2i+1)}(0)=x^{(2i+1)}(1)=0,\quad i=1,\ldots,n-1, \end{cases} \tag{6}$$