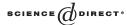


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Blowup behavior for a nonlinear parabolic equation of nondivergence form

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1. Introduction

We consider the following Cauchy problem for a nonlinear parabolic equation:

$$u_t = u(u_{xx} + u^p), \quad x \in (-\infty, \infty), t > 0,$$
 (1.1)

$$u(x, 0) = u_0(x), \quad x \in (-\infty, \infty),$$
 (1.2)

where p > 1 and $u_0(x) > 0$. We say that a solution u(x, t) blows up in a finite time T if $\sup_{x \in (-\infty, \infty)} u(x, t) \to \infty$ as $t \to T$.

It is well known that the solution to the system (1.1)–(1.2) blows up in a finite time T for certain $u_0(x)$. Suppose that u blows up at (0, T). To describe the blowup behavior near the point (0, T), we introduce the following similarity variables:

$$y = \frac{x}{(T-t)^{\beta}},\tag{1.3}$$

$$T - t = e^{-s}, \tag{1.4}$$

$$z(y,s) = (T-t)^{\alpha} u(x,t), \tag{1.5}$$

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where the similarity exponents are necessarily given by

$$\alpha = \frac{1}{p}, \quad \beta = \frac{p-1}{2p}.$$

Then u satisfies (1.1)–(1.2) if and only if z satisfies

$$z_s + \alpha z + \beta y z_v = z(z_{vv} + z^p), \quad y \in (-\infty, \infty), s > s_0 := -\ln T,$$
 (1.6)

$$z(y, s_0) = z_0(y) := T^{\alpha} u_0(T^{\beta} y), \quad y \in (-\infty, \infty).$$
(1.7)

Hence to study the blowup behavior of u near the blowup point (0, T) is equivalent to study the stabilization problem of (1.6) as $s \to \infty$. Note that the only nonzero constant solution of (1.6) is given by

$$\kappa := \alpha^{\alpha} = (1/p)^{1/p}. \tag{1.8}$$

Motivated by a work of Galaktionov [2], we shall describe the blowup behavior of the solution near the blowup point by constructing a Lyapunov function (see, e.g., [7]). To construct a suitable Lyapunov function, we need to study the stationary solution of (1.6):

$$\varphi_{yy} - \beta y \varphi^{-1} \varphi_y + \varphi^p - \alpha = 0. \tag{1.9}$$

Actually, to overcome the difficulties of possible singularity near the origin, we shall modify the ODE (1.9) near the origin. For such a modification, we also refer to [5] for a study on a quenching problem. The main difference between these two problems is the term in which the singularity appears. The singularity appears in the reaction term for the quenching problem in [5]. Here, we need to modify the singularity in the gradient term for our blowup problem.

We define the ω -limit set of the problem (1.6)–(1.7) by

$$\omega(z_0) = \{ \varphi \in C^2(\mathbb{R}) \mid \text{ there exists a sequence } s_j \to \infty \text{ such that } z(\cdot, s_j) \to \varphi(\cdot) \text{ as } j \to \infty \text{ uniformly on compact sets in } \mathbb{R} \}.$$

We state our main theorems as follows:

Theorem 1.1. Assume that $u_0(x)$ is smooth and

$$\begin{array}{ll} u_{0}(x) > 0 & \text{for } x \in (-\infty, \infty), \\ u_{0}''(x) + u_{0}^{p}(x) \geqslant 0 & \text{for } x \in (-\infty, \infty), \\ u_{0}(x) = u_{0}(-x) & \text{for } x > 0, \\ u_{0}'(x) \leqslant 0 & \text{for } x > 0, \ u_{0}'(x) \not\equiv 0, \end{array}$$

and that T is the blowup time. Then the ω -limit set of (1.6)–(1.7) is not empty and any ω -limit is a solution of the ODE (1.9) such that

$$\varphi_{\xi} \! \leqslant \! 0, \quad \varphi_{\xi\xi} + \varphi^p \! \geqslant \! 0 \quad for \ \xi \in [0, \infty).$$

Theorem 1.2. If we further assume

$$\lim_{x \to \infty} \inf x^{\alpha/\beta} u_0(x) = +\infty, \tag{1.10}$$

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