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Local approximation by Beta operators

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Abstract

The present paper deals with the Beta approximation operators. We obtain an estimate on the rate of convergence for functions of bounded variation by means of the decomposition technique. Furthermore we derive the complete asymptotic expansion of the sequence $((L_n f)(x))_{n=1}^{\infty}$ for smooth functions as n tends to infinity.
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1. Introduction

The Beta operators $L_n (n \in \mathbb{N})$ approximate Lebesgue integrable functions f on the interval $I = (0, 1)$ by

$$(L_n f)(x) = \frac{1}{B(nx, n(1-x))} \int_0^1 t^{nx-1}(1-t)^{n(1-x)-1} f(t)\, \mathrm{d}t, \tag{1}$$

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where $B(.,.)$ denotes the Beta function. It is easily verified that the operators are positive linear operators preserving linear functions.

The Beta operators were introduced by Lupas [6] in a slightly different form. Definition (1) was given by Khan [4].

In the present paper we study the rate of convergence of the operators L_n. The next section contains an estimate of $|(L_n f)(x) - \frac{1}{2}\{f(x+) + f(x-)\}|$ for functions f of bounded variation (see Theorem 1). This result partially improves an earlier estimate due to Khan [4]. We mention that in [5] Khan studied approximation of bounded variation functions by general classes of operators.

The last section presents the complete asymptotic expansion for the operators (1) for locally smooth functions. We derive the complete asymptotic expansion of the operators L_n as n tends to infinity (see Theorem 5). It turns out that the appropriate representation is a series of reciprocal factorials of the form

$$(L_n f)(x) \sim f(x) + \sum_{k=1}^{\infty} \frac{c_k(f,x)}{n^{\overline{k}}} \quad (n \to \infty), \tag{2}$$

provided that $f \in L^\infty(0,1)$ possesses derivatives of all orders at x. By $n^{\overline{k}} = n(n+1)\dots(n+k-1)$, $n^{\overline{0}} = 1$, we denote the rising factorial. All coefficients $c_k(f,x)$ are calculated explicitly. Formula (2) means that, for all $q \in \mathbb{N}$,

$$(L_n f)(x) = f(x) + \sum_{k=1}^{q} \frac{c_k(f,x)}{n^{\overline{k}}} + o(n^{-q})$$

as $n \to \infty$.

For the sake of a convenient notation in the proofs we rewrite the operators (1) as

$$(L_n f)(x) = \int_0^1 K_n(x,t) f(t)\,\mathrm{d}t, \tag{3}$$

where the kernel function K_n is given by

$$K_n(x,t) = \frac{t^{nx-1}(1-t)^{n(1-x)-1}}{B(nx, n(1-x))}. \tag{4}$$

2. Rate of convergence for functions of bounded variation

2.1. The result

Throughout this note, for fixed $x \in I$, we use the auxiliary function f_x, which is defined by

$$f_x(t) = \begin{cases} f(t) - f(x-) & (0 < t < x), \\ 0 & (t = x), \\ f(t) - f(x+) & (x < t < 1). \end{cases} \tag{5}$$

The following theorem is the main result of the present section.

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