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A new semi-analytical method for the analysis of tapered optical waveguides

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ABSTRACT

Tapered waveguide are used in number of integrated optic devices such as directional couplers, modulators, switches mode converters, etc. Most of the methods analyzing tapered waveguide are numerical in nature. In this paper we present a simple, fast and accurate semi analytical method for *z*-varying waveguide. However, very few idealized structures can be analyzed directly. The present method consists of separating transverse and longitudinal variation in the wave equation, leading to a differential equation with *z*-varying coefficients for the field variation along *z*-axis. For the transverse variation local normal theory is applied. Now this equation is applied to specific taper geometries like linear down taper. Computational are done assuming typical values. We observe variation of power mode profile and mode width. Waveguide loss is also including in the analysis. Finally the analytical simulation results have been verified by the commercial Opti-BPM software.

1. Introduction

Tapered waveguides form an integral part of a wide range of both passive and active components in modern opto-electronics. Interconnection losses between single mode integrated-optic devices and single mode fibers need to be small so that these devices can find application in practical system. Apart from being used as low loss connectors between waveguides of different dimension, they are also part of waveguide junctions and branches, and hence of a very wide range of integrated-optics fibers devices such as directional couplers, modulator, switches and mode converters [1,2]. It is necessary to analyze the propagation of electric field in the tapered waveguides for achieving desired result, but there are only a very few cases of optical field propagation in dielectric structures that admit to analytic solutions. The situation is even more acute if longitudinal non-uniformity is included. The most commonly used method for the analysis of the field propagation is probably the beam propagation method (BPM). Other numerical method such as finite difference (FD), finite element method (FEM), etc. is also used. These are robust, versatile and applicable to a wide variety of structures. Unfortunately, this is often achieved at the expenses of long computational times and large memory requirements, both of which can become critical issues especially when structures with large dimension are considered or when used within an interactive design environment. For avoiding these difficulties, semi-analytical methods such as Marcatillis method, effective index method etc, are used as an alternative approach to the numerical method [3–7]. Since these are very efficient, often provide accuracy comparable with that of numerical methods and are also easily implemented, they are still highly valuable for the design of a particular devices or even entire circuits. But each semi-analytical method is usually limited to a certain type or class of problem.

In this section we present a simple, fast and accurate semi-analytical method for the analysis of optical field propagation in two dimensional (2D) linear tapered waveguide. First part of this section shows the computational work where characterized are studied by assuming some typical values. In this part first we determine the variation of propagation constants then the variation of power and mode width along with power loss along the taper direction is studied [8–20].

2. A new semi-analytical method

We consider a two dimensional step index linear tapered waveguide of which *xz* view is shown in Fig. 1. This structure is assumed to be infinitely long in *y*-direction.

$\rho(z)$ is the half width of tapered waveguide and is given by the following equation:

$$\rho(z) = \frac{a_f - a_i}{2L} z + \frac{a_i}{2} \tag{1}$$

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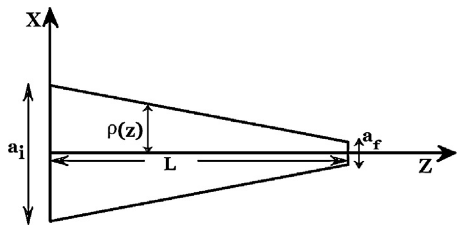

Fig. 1. 2-D linear tapered waveguide.

where a_f and a_i represent the final full width, respectively and L is the length of waveguide in z-direction. Let the refractive index distribution $n\,(x,z)$ is given by

$$\left.\begin{aligned} n(x,z) &= n_1 \quad \text{when} \quad |x| \le \rho(z) \\ n(x,z) &= n_2 \quad \text{when} \quad |x| \ge \rho(z) \end{aligned}\right\} \tag{2}$$

Now, we consider electric field distribution in 2-D waveguide given by

$$\underset{\sim}{E} = E(x,z)e^{j(\omega t-\beta z)} \tag{3}$$

where β the z-component of wave is vector and ω is the angular frequency of the electric field. In terms of electric field components, we can write

$$\underset{\sim}{E}_j = E_j(x,z)e^{j(\omega t-\beta z)} \quad \text{where} \quad j = x, y, z. \tag{4}$$

By using Maxwell's equation, wave equation for TE modes can be written as

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} - 2j\beta\frac{\partial E_y}{\partial z} + (k_0^2n^2-\beta^2)E_y = 0 \tag{5}$$

For solving above Eq. (5), we assume that transverse component $E_y(x,z)$ of electric field $E(x,z)$ can be written as multiplication of two functions $\eta(x)$ and $\varphi(z)$ i.e.

$$E_y(x,z) = \eta(x)\varphi(z) \tag{6}$$

On substituting Eq. (6) in Eq. (5), we get

$$\frac{1}{\eta}\frac{\partial^2\eta}{\partial x^2} + (k_0^2n^2-\beta^2) = -\frac{1}{\varphi}\left[\frac{\partial^2\varphi}{\partial z^2} - 2j\beta\frac{\partial\varphi}{\partial z}\right] \tag{7}$$

Now, we use the mode property which gives (by using local normal mode theory)

$$\frac{\partial^2\eta}{\partial x^2} + (k_0^2n^2-\beta^2)\eta = 0 \tag{8}$$

Then, Eq. (7) becomes

$$\frac{\partial^2\varphi}{\partial z^2} - 2j\beta\frac{\partial\varphi}{\partial z} = 0 \tag{9}$$

Eq. (8) in two different regions $|x| \le \rho(z)$ and $|x| \ge \rho(z)$ can be written as

$$\frac{\partial^2\eta}{\partial x^2} + (k_0^2n_1^2-\beta^2)\eta = 0 \quad \text{when} \quad |x| \le \rho(z) \tag{10a}$$

$$\frac{\partial^2\eta}{\partial x^2} + (k_0^2n_2^2-\beta^2)\eta = 0 \quad \text{when} \quad |x| \ge \rho(z) \tag{10b}$$

Since we have considered symmetric tapered so the solution of Eq. (10) are given by [2]

$$\eta(x) = Ce^{(-\gamma|x|)} \quad \text{when} \quad |x| \ge \rho(z) \tag{11a}$$

$$\eta(x) = A\cos(\kappa x) \quad \text{when} \quad |x| \le \rho(z) \quad \text{symmetrical mode} \tag{11b}$$

$$\eta(x) = B\sin(\kappa x) \quad \text{when} \quad |x| \le \rho(z) \quad \text{antisymmetrical mode} \tag{11c}$$

$$\eta(x) = \frac{x}{|x|}De^{(-\gamma|x|)} \quad \text{when} \quad |x| \ge \rho(z) \tag{11d}$$

where A, B, C and D are constants, $\kappa = \sqrt{\kappa_0^2n_1^2-\beta^2}$ and $\gamma = \sqrt{\beta^2-\kappa_0^2n_2^2}$.

Boundary condition that $\eta(x)$ and $\partial\eta/\partial x$ are continuous a $tx = \pm\,\rho(z)$, when applied to Eq. (11) gives

$$\tan\ k\rho(z) = \frac{\gamma}{\kappa} \quad \text{for symmetric modes} \tag{12a}$$

$$\tan\ k\rho(z) = -\frac{\kappa}{\gamma} \quad \text{for antisymmetric modes} \tag{12b}$$

These equations are transcendental equation in κ and the solution of these equations gives the allowed value of β. For studying the characteristics of the fundamental modes, which is a symmetrical mode, we consider dispersion equation (12a). By using Eq. (1), dispersion equation (12) for symmetric modes can be written as

$$z = \frac{1}{\kappa t_1}\tan^{-1}\left(\frac{\gamma}{x}\right) - \frac{l_1}{t_1} \tag{13}$$

This equation gives the variation of propagation constant β along the propagation direction for the propagation direction for given value of t_1, l_1, n_1 and n_2. For different taper angles (tangents of t_1 gives the taper angle) corresponding variation of β can be obtained. After obtaining the variation of propagation constant β with respect to z, we substitute $\beta(z)$ in Eq. (9) which is then solved for obtaining $\varphi(z)$. $\varphi(z)$, in general, is a complex function so it affects the amplitude as well as phase of the electric field. Let $\varphi_r(z)$ and $\varphi_i(z)$ are the real and imaginary part of the $\varphi(z)$, then $\varphi(z)$ will be given by

$$\varphi(z) = \varphi_r(z) + \varphi_i(z) \tag{14}$$

Now, the value of $\eta(x)$ and $\varphi(z)$ is substituted in Eq. (6) and finally $E_y(x,z)$ in Eq. (3), we get the total electric field distribution in step index linear tapered waveguide.

$$P = \frac{1}{2}\int Re\left\langle \underset{\sim}{E} \times \underset{\sim}{H}^*\right\rangle .\hat{z}ds. \tag{15}$$

where s is the surface area of transverse cross section and z is the direction of the waveguide axis, normal to the surface s, and '*' denotes the complex conjugate. Then,

$$P(z) = \frac{1}{2\omega\mu_0}\left[\beta(\varphi_r^2+\varphi_i^2) + \varphi_i\frac{\partial\varphi_r}{\partial z} - \varphi_r\frac{\partial\varphi_i}{\partial z}\right]2\int_0^\infty \left|\eta(x)\right|^2dx. \tag{16}$$

This equation gives the power per unit length in y-direction. After substituting the value of $\eta(x)$ from Eq. (11) the power per unit length in y-direction in the core of the waveguide will be given by

$$P(z) = \frac{|A|^2}{2\omega\mu_0}\left[\beta(\varphi_r^2+\varphi_i^2) + \phi_i\frac{\partial\varphi_r}{\partial z} - \varphi_r\frac{\partial\varphi_i}{\partial z}\right]\left[\rho(z)+\frac{1}{\gamma}\right]. \tag{17}$$

Here A is an unknown constant which can be calculated by using the fact that at the input of the waveguide i.e. at $z = 0$ the launched power is $p(0)$ then from the above equation we get

$$P(0) = \frac{|A|^2}{2\omega\mu_0}\beta(z=0)\left[\rho(z=0)+\frac{1}{\gamma(z=0)}\right]$$
$$\text{where} \quad \rho(z=0) = \frac{a_i}{2} \tag{18}$$

Eq. (18) gives the value of the constant A. This value of A is substituted in Eq. (17), which then gives the power variation in fundamental mode with respect to z in linear tapered waveguide. From the knowledge of the value of A, one can see the variation of modes profile and modes along z. The MATLAB simulation of mode profile along the z-direction can be represented in Fig. 2. Finally this analytical result has been verified by commercial software.