Optik 127 (2016) 8710–8727

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

A fractional order variational model for the robust estimation of optical flow from image sequences

Pushpendra Kumar [a,*], Sanjeev Kumar [a], Balasubramanian Raman [b]

[a] Dept. of Mathematics, Indian Institute of Technology Roorkee, Roorkee 247667, India
[b] Dept. of Computer Science & Engineering, Indian Institute of Technology Roorkee, Roorkee 247667, India

ARTICLE INFO

Article history:
Received 5 August 2015
Accepted 26 May 2016

Keywords:
Fractional derivative
Image sequence
Optical flow
Partial differential equations
Variational methods

ABSTRACT

In this paper, a fractional order variational model for estimating the optical flow is presented. In particular, the proposed model generalizes the integer order variational optical flow models. The fractional order derivative describes discontinuous information about texture and edges, and therefore a more suitable in estimating the optical flow. The proposed variational functional is a combination of a global model of Horn and Schunck and the classical model of Nagel and Enkelmann. This formulation yields a dense flow and preserves discontinuities in the flow field and also provides a significant robustness against outliers. The Grünwald–Letnikov derivative is used for solving complex fractional order partial differential equations. The corresponding linear system of equations is solved by an efficient numerical scheme. A detailed stability and convergence analysis is given in order to show the mathematical applicability of the numerical algorithm. Experimental results on various datasets verify the validity of the proposed model.

1. Introduction

Recovery of optical flow from a sequence of images is a key problem in computer vision and image processing. Optical flow is defined as a 2*D* velocity vector which arises either due to the motion of the objects in the scene or by the motion of the observer/camera. The objective of the problem is to determine the displacement in terms of optical flow between two image frames. Therefore, it is known as correspondence problem in vision system. Optical flow furnishes the dynamic information of an object between the image frames of a scene, i.e., how many units the pixel/object has moved compared to the previous frame. It has a much higher level applications in vision system such as 3*D* reconstruction, automatic navigation, video surveillance, human action understating and medical diagnosis [1–4]. Extraction of the optical flow has been also used as a preprocessing step in many vision algorithms such as visual control, motion parameter estimation and image segmentation [5–8]. Estimation of the flow field is considered as an ill-posed problem. Thus, it is required to consider some additional constraints in order to regularize the flow field during optical flow estimation. Many models have been proposed to determine the optical flow in the literature starting from the seminal work [9,10] and achieved an impressive level of accuracy. A global approach proposed by Horn and Schunck [9] yields a dense flow, but it is experimentally sensitive to noise. A local approach proposed by Lucas and Kanade [10] is robust under noise, but it is unable to yield dense flow.

* Corresponding author.
E-mail address: pushpdma@iitr.ac.in (P. Kumar).

http://dx.doi.org/10.1016/j.ijleo.2016.05.118

In the recent past, differential variational models are one of the most successful approaches for the estimation of optical flow and therefore achieved a good attention of researchers. The reason behind it is their simplicity and advantages in modeling, estimation and quality [11–15]. In order to improve the estimation accuracy of the optical flow models, many assumptions have been made in the variational models to get a significantly better performance. Recent models like [11,12,15–18] include the additional constraints such as gradient constancy assumption and the convex robust data term to obtain the impressive level of performance under outliers, noise and to avoid the problem of local minima. Many of the variational models such as [11–13,19–23] considered different constraints or regularizers to get piecewise optical flow and preserve discontinuity at the object boundaries. In [12,17], the variational functionals were proposed to estimate the optical flow for large motion. Moreover, some researchers have proposed the motion segmentation or parametric model to break the motion field into several smooth and piecewise parts [11,20,24,25]. Nevertheless, these models still have lacked to yield dense and discontinuity preserving optical flow together due to the inherent characteristics of the parametric models. All these models are based on integer order differentiation. These integer order differential variational models can be generalized from integer order to fractional order and categories into the class of fractional order variational models. The fractional order differential based methods are now more popular in image processing applications such as texture analysis, image de-blurring and image restoration [26–34].

The core idea of fractional order differentiation was introduced at the end of sixteen century and later published in the nineteenth [35,36]. The fractional order differential models deal with differentiation of arbitrary order and therefore known by the generalization of integer order variational models. In the literature, numerous authors have been proposed the definitions of fractional order differentiation. But, the quite popular fractional order differentiations among them are Caputo definition [37], Grünwald–Letnikov definition [38] and Riemann–Liouville definition [39]. The salient difference between the fractional and integer order differentiations is that we can find the fractional derivatives even if the function is not continuous whereas integer order derivative failed. Thus, fractional order derivatives efficiently provides the discontinuous information about texture and edges in the optical flow field. Due to non-local property, fractional order derivatives give an excellent tool for the description of memory and modeling of many characteristic phenomena in engineering and sciences. Moreover, fractional order differentiations provide their optical fractional order corresponding to a stable solution.

1.1. Contribution

In this paper, we proposed a fractional order variational model to estimate the optical flow from a sequence of images. The novelty of the proposed model compared to previous approaches is in three-fold. First, the proposed model generalizes the existing variational models from integer order to fractional order. Second, the given variational functional is formed from a global model of Horn and Schunck [9] and the classical model of Nagel and Enkelmann [13]. Thus, it combined the advantages of each of them. This helps to lead a dense flow over a region and preserves discontinuities in the optical flow. But, the recent model [28] is based on [9], which provides only dense flow and unable to preserve discontinuities. Third, due to the presence of fractional order derivatives, the proposed model efficiently handles texture and edges. The related Euler–Lagrange equations are obtained corresponding to the fractional order variational model. The numerical discretization of the complex fractional order partial differential equations is based on Grünwald–Letnikov derivative and solved by an efficient numerical scheme. The performance of the proposed model has been tested on various spectrum databases, and compared with some recently published techniques such as [28,40]. The convergence analysis of the used numerical scheme is provided to support the applicability of the algorithm. The robustness of the proposed model are shown under a considerable amount of noise.

The rest of the paper is organized in the following sections: In Section 2 some preliminaries related to the fractional order differentiations have been given. Section 3 describes the optical flow constraints followed by proposed fractional variational model for optical flow estimation. Section 4 describes the numerical scheme with its convergence analysis and a pseudo code of the algorithm. Section 5 describes the experimental datasets and evaluation methods followed by the experimental results. Conclusions are given in Section 6.

2. Mathematical preliminaries of fractional order derivatives

2.1. Riemann–Liouville definition

Let $g(t) \in L^1([a, b])$ be a function and α be a fractional value between $m-1$ to m, $m \in \mathbb{Z}^+$. The left Riemann–Liouville fractional integral of order α is defined as [39]

$$ {}_aI_t^{\alpha} g(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\gamma)^{\alpha-1} g(\gamma) d\gamma, \quad t \in [a, b] \tag{1} $$

where $\Gamma(\alpha)$ represents the gamma function, which can be defined as an improper integral

$$ \Gamma(\alpha) = \int_0^{\infty} e^{-t} t^{\alpha-1} dt \tag{2} $$