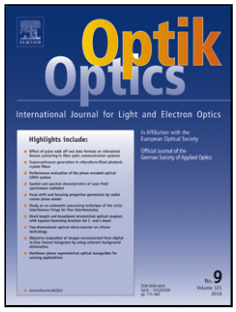

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RATIONAL APPROXIMATION OF SOME LIMITED SMOOTH FUNCTIONS

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Abstract. In this paper, Newman-type raional interpolation to some limited smooth functions induced by some set of nodes X in $[-1,1]$ is considered and the upper and lower bound of the order of approximation is given, which is sharper than Stahl's result.And the error estimation is

$$Cn^{-\frac{\alpha^2}{\alpha+1}}\exp(-n^{\frac{\alpha}{\alpha+1}}) \le |e_n(X;x)| \le 3\exp(-n^{\frac{\alpha}{\alpha+1}}).$$

1. Introduction

The Helmholtz equation, which represents a time-independent form of the wave equation, results from applying the technique of separation of variables to reduce the complexity of the analysis. Some numerical methods such as Padé rational approximation method, Marching and bidirectional beam propagation method which are used to solve Helmhotlz equation have been studied by many scholars. Padé "Parabolic" approximation of (1.1) consists in formally approximating the pseudo-differential square root operator $\sqrt{1-L}$ by rational functions of L [1]:

$$\sqrt{1-L} \approx \frac{p_0 - p_1\lambda + p_2\lambda^2 - \cdots + p_l\lambda^l}{1 - q_1\lambda + q_2\lambda^2 - \cdots + q_m\lambda^m} =: \frac{P_l(\lambda)}{Q_m(\lambda)} \tag{1.1}$$

Newman [2] demonstrated that rational approximation to some funtions is much more favorable in contrast to polynomial approximation. It may be appropriate to repeat a remark from Gonchar [3], where it was pointed out that Newman's result can be obtained rather immediately from an old result(from 1877) by Zolotarev. In this sense, the investigation of rational approximation dates back even further than Bernstein's work on polynomial approximation. However, it is impartiality to say that Newman's study generated a great deal of research on rational approximation of some non-smooth functions or some smooth functions of a finite degree. Although our research is also motivated by the result of Newman, we generalize the approximated function and

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