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Fast bilateral filtering using the discrete cosine transform and the recursive method☆

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ABSTRACT

A variety of fast bilateral filtering algorithms have been proposed in recent years. We found that many of them try to decompose the bilateral filter into a series of spatial filters through range kernel decomposition. This paper presents a uniform framework for algorithms in this category. Within this framework, a new accelerating algorithm for bilateral filter is proposed. The proposed algorithm decomposes the range kernel using discrete cosine transform (DCT) and implements the spatial filtering using the recursive method. Experimental results demonstrate that the proposed algorithm outperforms current state-of-the-art algorithms.

1. Introduction

The bilateral filter [2] is a non-linear filter, which smoothes out an image by averaging neighborhood pixels, but preserves sharp edges by decreasing weights when intensity difference is large. The bilateral filter is widely used in many image processing applications. However, the direct implementation of a bilateral filter turns out to be rather computational intensive. As a result, a variety of accelerating algorithms [1,3–17] have been proposed recent years.

Paris and Durand [7] expressed the bilateral filter as a high-dimensional Gaussian convolution. The high-dimensional Gaussian convolution is then accelerated by bilateral grid [18], Gaussian KD-Trees [6], permutohedral lattice [9] or adaptive manifolds [19]. Pham and van Vliet [4] approximated the 2D bilateral filter by first filtering the image rows and then filtering the columns. The idea of separable filtering has been pushed further by Yang in his recursive bilateral filter [20].

Durand and Dorsey [3] developed a piecewise-linear model to express bilateral filter as a group of spatial filters on intermediate images. After that, Taylor Series [5], Raised Cosines (RC) [10,11,13] have been used to construct intermediate images. And Fast Fourier Transform (FFT) [3], the integral image [5,12,14], and the recursive method [8] have been used to accelerate spatial filtering. Although these algorithms [1,3,5,8,10–17] were studied from different points of view in the original literatures, we found that there is a uniform framework behind them. All of them try to convert the bilateral filter into a series of spatial filters on the intermediate images through range kernel decomposition.

The contributions of this paper are twofold. On one hand, this paper presents a uniform framework for fast bilateral filtering algorithms based on range kernel decomposition, which may inspires fresh thinking and provides a new starting point for future research. On the other hand, within the proposed framework, this paper proposes a new fast bilateral filtering algorithm, which outperforms current state-of-the-art algorithms, such as RC and Cosine Integral Image (CII).

This paper is organized as follows. In Section 2, we briefly review related works and present our framework. In Section 3, a novel accelerating algorithm based on DCT and recursive method is proposed. Experimental evaluations and comparisons are presented in Section 4. At last, conclusions are drawn in Section 5.

2. The framework for fast bilateral filtering based on range kernel decomposition

2.1. Bilateral filter

The general form of a bilateral filter, which is also known as a joint bilateral filter [21], can be expressed as

$$\tilde{f}(x) = \eta^{-1} \int_{\Omega} \omega(\mathbf{x}, \mathbf{y}) \phi(g(\mathbf{x}), g(\mathbf{y})) f(\mathbf{y}) d\mathbf{y}, \quad (1)$$

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where

$$\eta = \int_{\Omega} \omega(\mathbf{x}, \mathbf{y})\phi(g(\mathbf{x}), g(\mathbf{y}))d\mathbf{y}. \quad (2)$$

In this equation, the *spatial kernel* $\omega(\mathbf{x}, \mathbf{y})$ measures the distance between pixel $\mathbf{x}$ and a nearby pixel $\mathbf{y}$. The *range kernel* $\phi(g(\mathbf{x}), g(\mathbf{y}))$ measures the intensity differences between these two pixels. f is the input image and g is the guidance image. This paper considers the most widely used bilateral filter where both ω and ϕ are Gaussian function. The standard deviation of the spatial and the range kernel is referred to as σ_s and σ_r respectively. In the rest of this paper, we reformulate the bilateral filter as

$$\tilde{f}(x) = \int_{\Omega} \omega(\mathbf{x} - \mathbf{y})\phi(g(\mathbf{x}) - g(\mathbf{y}))f(\mathbf{y})d\mathbf{y}. \quad (3)$$

In this equation, the normalization η is omitted for simplicity. η can be calculated similarly to $\tilde{f}(x)$ by setting $f(\mathbf{y}) \equiv 1$.

2.2. Framework for acceleration

The bilateral filter is non-linear due to the existence of the range kernel ϕ. However, when the range kernel can be decomposed as

$$\phi(g(\mathbf{x}) - g(\mathbf{y})) \approx \sum_{i=1}^{N} c_i(g(\mathbf{x}))\tau_i(g(\mathbf{y})), \quad (4)$$

the bilateral filtered response can be approximated by

$$\begin{aligned} \tilde{f}(x) &\approx \int_{\Omega} \omega(\mathbf{x} - \mathbf{y})\sum_{i=1}^{N} c_i(g(\mathbf{x}))\tau_i(g(\mathbf{y}))f(\mathbf{y})d\mathbf{y} \\ &\approx \sum_{i=1}^{N} c_i(g(\mathbf{x})) \int_{\Omega} \omega(\mathbf{x} - \mathbf{y})\tau_i(g(\mathbf{y}))f(\mathbf{y})d\mathbf{y}. \end{aligned}$$

In this way, the bilateral filtering process can be divided into three phases.

Firstly, construct the intermediate images as

$$h_i(\mathbf{x}) = \tau_i(g(\mathbf{x}))f(\mathbf{x}). \quad (5)$$

Secondly, compute a spatial filter for each intermediate image as

$$\tilde{h}_i(x) = \int_{\Omega} \omega(\mathbf{x} - \mathbf{y})h_i(\mathbf{y})d\mathbf{y}. \quad (6)$$

Thirdly, weighted sum up $\tilde{h}_i(\mathbf{x})$ as

$$\tilde{f}(\mathbf{x}) \approx \sum_{i=1}^{N} c_i(g(\mathbf{x}))\tilde{h}_i(\mathbf{x}). \quad (7)$$

In this way, the bilateral filtered response is expressed as the weighted sum of the spatial filtered responses on the intermediate images. Fig. 1 presents the pseudo code for accelerating bilateral filter using Eqs. (4)–(7). It can be viewed as a uniform framework for algorithms in [1,3,5,8,10–17]. These algorithms differ in the ways they decompose the range kernel and implement the spatial filtering.

2.3. Range kernel decomposition

The local linear model [3] is a valid instance of range kernel decomposition, since it can be expressed as

$$\phi(g(\mathbf{x}) - g(\mathbf{y})) = \sum_{i=1}^{N} \delta(g(\mathbf{x}) - i)\phi(i - g(\mathbf{y})), \quad (8)$$

Algorithm 1 Fast bilateral filtering based on range kernel decomposition

```
f(x) : input image
g(x) : guidance image
c_i, τ_i, N : parameters in equation (4)
begin
  f̃(x) = 0
  for i = 1 to N do
    h_i(x) = τ_i(g(x)) f(x)
    h̃_i(x) = spatial_filtering(h_i(x))
    f̃(x) = f̃(x) + c_i(g(x)) h̃_i(x)
  end for
end
```

Fig. 1. Pseudo code for fast bilateral filtering based on range kernel decomposition.

where i is all the possible intensities of the guidance image and δ is the impulse function. In practice, the guidance image is usually subsampled in the range domain to reduce N [3,8,15]. The local histograms of the guidance image, which can be efficiently computed using the integral histogram [22], is widely used to implement the local linear model [5,12,14,16,17].

Porikli [5] pointed out that polynomials range kernel can be expressed as Eq. (4) and arbitrary range kernel can be approximated using Taylor polynomials. Chaudhury [10,11,13] proposed that cosine and sine functions can also be expressed as Eq. (4), and a Gaussian range kernel can be approximated by RC. This group of decompositions is generalized to shiftable function theory in [10]. Recently, Elboher and Werman [1] used DCT to decompose the filter kernel to build CII. Inspired by their work, we study range kernel decomposition using DCT in Section 3.

2.4. Fast spatial filtering

There are many ways to accelerate spatial filtering. Spatial filtering can be converted to multiplication in frequency domain using FFT. But FFT has a $O(\log(r))$ complexity, where r is the filter size. The box spatial filter can be implemented with a $O(1)$ complexity using the integral image, which is also known as summed-area tables [23]. Other kinds of spatial filtering can be accelerated using *repeated integration* [24], *kernel integral image* [25] or *CII* [1]. Another spatial filtering algorithm with $O(1)$ complexity is the recursive method [26]. According to [1], recursive method is currently the fastest implementation for Gaussian spatial filtering.

2.5. Analysis

The framework described in Fig. 1 approximates a bilateral filter using N spatial filters. As a result, the run time of bilateral filtering depends on N and the spatial filter implementation. Besides, approximation accuracy in Eq. (4) and the spatial filter implementation will determine the accuracy of the filter output $\tilde{f}(\mathbf{x})$. This framework also reveals that the way to implement the spatial filtering is independent on the way to decompose the range kernel. Thus they can be studied independently. And the first-class range kernel decomposition algorithm and the first-class fast spatial filtering algorithm can be directly combined into a first-class fast bilateral filtering algorithm.

3. Fast bilateral filtering using DCT and recursive method

In this section, a novel accelerating algorithm for bilateral filtering using DCT and recursive method is proposed. Our idea is to combine the currently best-performance range kernel decomposition algorithm with the currently best-performance fast spatial filtering algorithm.