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Total reflection in a uniaxial crystal–uniaxial crystal interface

María C. Simon*, Karina B. Bastida, Karin V. Gottschalk

Departamento de Física, Laboratorio de Optica, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria –Pabellón I (1428) Buenos Aires, Argentina

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Abstract

Considering an interface between two uniaxial birefringent crystals, four reflected and four refracted waves for each incidence direction are obtained. Along this direction can propagate an ordinary wave and an extraordinary wave. Here, we present the analytic expressions for the four critical angles, from which each refracted wave no more exists as propagating wave. We show the variation in these critical angles for different interfaces varying the orientation of the incidence plane with respect to the optical axes. When both principal refractive indices of the second medium are smaller than those of the first medium, then the four critical angles exist for each incidence plane and for any direction of the optical axes. But, when one of the indices has an intermediate value between the values of the indices of the other crystal, we can chose the optical axes directions in such a way that certain critical angles do not exist. Therefore, we can select the refracted wave that is eliminated by total reflection.

Keywords: Birefringence; Total reflection; Critical angle

1. Introduction

Reflection and refraction of monochromatic waves at interfaces, for which one of the media is a birefringent one, have been studied from different kind of view and with different objectives.

The first problem at an interface between an isotropic medium and a uniaxial anisotropic medium is the calculus of the propagation direction of the extraordinary refracted ray. Many authors have solved this problem in different ways. [1–4]

The total reflection phenomenon, that is very simple for interfaces between isotropic media, becomes very difficult when one of the media is a birefringent one. When light is incident from an isotropic medium on to the surface of a birefringent crystal, two refracted rays are obtained. Consequently, two critical angles exist at which each refracted wave ceases existing as a propagating way. The variation in these critical angles with the incident direction for different optical axes directions has been described in details for uniaxial crystals [5,6] and biaxial crystals [7].

When the first medium is a birefringent one, two critical angles are also obtained: one for the ordinary incident ray and the other one for the extraordinary incident ray [8]. For these interfaces in which the first medium is birefringent, two reflected rays for each incident ray exist, and the reflection is asymmetric (excepting the case in which the incident and the reflected wave are ordinary waves). Because of the asymmetry of the reflection, there is a critical angle for the reflected ray, this is, there is inhibited reflection [9].

*Corresponding author.
E-mail address: hohn@netizen.com.ar (M.C. Simon).

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In the present paper, we analyze the total reflection phenomenon at an interface between two uniaxial crystals. As we have two different possibilities for the incident wave (it can be an extraordinary or an ordinary wave), four refracted rays will exist. This gives rise to four critical angles that depend on the parameters characterizing the interface.

2. Characteristics of the interface

Two uniaxial crystals form the interface. Light is incident from the first crystal whose principal refractive indices are n_{o1} and n_{e1}. The following relations give the respective principal phase velocities:

$$u_{o1} = \frac{c}{n_{o1}}; \quad u_{e1} = \frac{c}{n_{e1}}, \tag{1}$$

c being the light velocity in vacuum. The principal refractive indices and the principal phase velocities of the second crystal are:

$$u_{o2} = \frac{c}{n_{o2}}; \quad u_{e2} = \frac{c}{n_{e2}}. \tag{2}$$

The optical axes directions are given by the unit vectors $\check{\zeta}_3$ (first crystal) and $\check{z}_3$ (second crystal). Defining the coordinate system of the interface by means of the unit vectors $\check{x}, \check{y}, \check{z}$ and the angles θ_1, θ_2 and ϕ as in Fig. 1a, we have:

$$\check{\zeta}_3 = -\check{x}\sin(\theta_1) + \check{z}\cos(\theta_1), \tag{3}$$

$$\check{z}_3 = -\check{x}\sin(\theta_2) + \check{y}\cos(\theta_2)\sin(\phi) + \check{z}\cos(\theta_2)\cos(\phi). \tag{4}$$

The incident wave can be ordinary or extraordinary. In both cases, the unit vector normal to the wavefront together with the unit vector normal to the interface ($\check{x}$) defines the incidence plane. This plane forms an angle δ with the ($\check{x}, \check{z}$) plane. The unit vector normal to the ordinary incident wavefront can be expressed as a function of δ angle and the incidence angle α_o in this way:

$$\check{N}_o = \check{x}\cos(\alpha_o) + \check{y}\sin(\alpha_o)\sin(\delta) + \check{z}\sin(\alpha_o)\cos(\delta). \tag{5}$$

The unit vector normal to the extraordinary incident wavefront is:

$$\check{N}_e = \check{x}\cos(\alpha_e) + \check{y}\sin(\alpha_e)\sin(\delta) + \check{z}\sin(\alpha_e)\cos(\delta). \tag{6}$$

The phase velocity of the ordinary incident wave is u_{o1} for any $\check{N}_o$ direction. On the other hand, the phase velocity of the extraordinary wave depends on the propagation direction of the wave $\check{N}_e$ according to: [3]

$$(u'')^2 = \frac{c^2}{n_{e1}^2 n_{o1}^2}\left\{n_{o1}^2 + (n_{e1}^2 - n_{o1}^2)(\check{N}_e \cdot \check{\zeta}_3)^2\right\}. \tag{7}$$

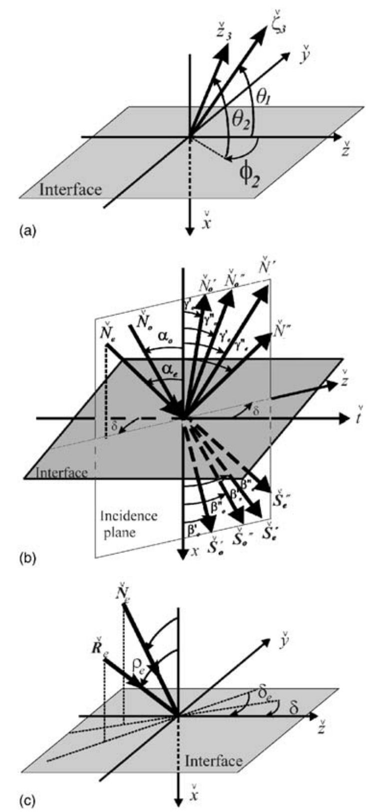

Fig. 1. Coordinate system: (a) directions of the normals to the extraordinary and ordinary incident, reflected and refracted wavefronts; (b) normal wavefront-ray direction relationship for the incident extraordinary wavefront; (c) the optical axes directions.

The energy flux direction, this is, the direction of the light ray, can be obtained by means of the following vectorial equation:

$$\check{R}_e = \frac{1}{f_e}\left\{n_{o1}^2\,\check{N}_e + (n_{e1}^2 - n_{o1}^2)(\check{N}_e \cdot \check{\zeta}_3)\,\check{\zeta}_3\right\}, \tag{8}$$

where f_e is a normalization factor given by:

$$(f_e)^2 = n_{o1}^4 + (n_{e1}^4 - n_{o1}^4)(\check{N}_e \cdot \check{\zeta}_3)^2. \tag{9}$$

From these relations, we can see that the unit vector along the ray direction $\check{R}_e$ is not generally contained in the incidence plane. Defining the angles ρ_e and δ_e as in Fig. 1(b), we have:

$$\check{R}_e = \check{x}\cos(\rho_e) + \check{y}\sin(\rho_e)\sin(\delta_e) + \check{z}\sin(\rho_e)\cos(\delta_e). \tag{10}$$

For each ordinary or extraordinary incidence ray, two reflected rays and two refracted rays are obtained. Therefore, there are four critical angles. We are going to treat each case separately.