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Propagation of Bessel-modulated Gaussian beams through a paraxial ABCD optical system with an annular aperture

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Abstract

Based on the generalized Collins diffraction integral and the expansion of the hard aperture function into a finite sum of complex Gaussian functions, the approximate analytical expressions of Bessel–Gaussian beams and QBG beams passing through a paraxial ABCD optical system with an annual aperture are derived. As special cases, the corresponding closed-forms for the unaperture or circular aperture or circular black screen cases are also given. The results provide more convenience for studying their propagation and transformation than the usual way by using diffraction integral formula directly. Numerical examples are given to illustrate the propagation properties of Bessel–Gaussian and QBG beams.

Keywords: Bessel–Gaussian beams; QBG beams; ABCD optical system; Annular aperture

1. Introduction

The propagation of paraxial light beams in optical systems is of great importance and attracts the attention of many optical researchers for a long time, and various closed-form paraxial solutions of the Helmholtz equation have been described over the past years. A particularly interesting class of such solutions, known as Bessel–Gaussian beams, has been introduced by Gori and co-workers in 1987 [1]. In 1999, Caron and Potvliege proposed another class of simple closed-form solution of the paraxial wave equation, which will be referred to as 'QBG beams' in the following [2]. The QBG beams is similar to the Bessel–Gaussian beams, however, the argument of the Bessel function is not linear in the transverse coordinate but quadratic. Recently, a large number of papers have been devoted to studying the propagation characteristics of Bessel-modulated Gaussian light beams [3–10]. Up to now, the Bessel-modulated Gaussian light beams passing through a paraxial ABCD optical system with an annular aperture, to our knowledge, has not been studied elsewhere. In fact, the annular apertured case represents the more general case, and the unapertured or apertured or black screen case could be regarded as its special case. More recently, the propagation of Laguerre–Gaussian beams through a paraxial ABCD optical system with an annular aperture has been analyzed based on the generalized Huygens–Fresnel diffraction integral and the expansion of the hard-edged aperture function into a finite sum of complex Gaussian functions [11]. Using the similar way as for Laguerre–Gaussian beams, the propagation of Bessel-modulated Gaussian light beams

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passing through a paraxial ABCD optical system with an annular aperture will be studied in this paper.

The paper is organized as follows. The propagation and transformation of Bessel-modulated Gaussian light beams passing through a paraxial ABCD optical system with an annular aperture is analyzed in Section 2. This analysis will be discussed in the two cases: Bessel–Gaussian and QBG beams. In Section 3, the applications of the propagation formulae are illustrated with detailed numerical examples, and a comparison with the method used diffraction integral directly is made, which shows the advantage of our approach. Finally, a simple conclusion is outlined in Section 4.

2. Approximate propagation equation of Bessel–Gaussian or QBG beams through an annular apertured paraxial ABCD optical system

Consider a paraxial optical system with an annular aperture illuminated by a Bessel–Gaussian or QBG light beam, which is represented by $E(r_0, \theta_0, z=0)$ at the input plane and by $E(r, \theta, z)$ at the output plane. For an optical system described by an ABCD matrix, the relationship (called also the Collins diffraction integral) between the input and output functions can be established as [12]

$$E(r,\theta,z) = \frac{\mathrm{i}k}{2\pi B}\int_0^{2\pi}\int_b^a E(r_0,\theta_0,0) \times\exp\left\{-\frac{\mathrm{i}k}{2B}[Ar_0^2 - 2rr_0\cos(\theta-\theta_0) + Dr^2]\right\} r_0\,\mathrm{d}r_0\,\mathrm{d}\theta_0, \tag{1}$$

where $k = 2\pi/\lambda$ is the wave number, a and b denote the out-radius and in-radius of the annular aperture, A, B, C and D are the elements of the ray matrix. In Eq. (1) an unimportant phase factor is omitted for the sake of simplicity.

Introducing the hard aperture function

$$A_{\mathrm{pa}}(r) = \begin{cases} 1, & |r| \leqslant a, \\ 0, & |r| > a, \end{cases} \quad A_{\mathrm{pb}}(r) = \begin{cases} 1, & |r| \leqslant b, \\ 0, & |r| > b, \end{cases} \tag{2}$$

then Eq. (1) becomes

$$E(r,\theta,z) = \frac{\mathrm{i}k}{2\pi B}\left\{\int_0^{2\pi}\int_0^{\infty} A_{\mathrm{pa}}(r_0)E(r_0,\theta_0,0) \times\exp\left\{-\frac{\mathrm{i}k}{2B}[Ar_0^2 - 2rr_0\cos(\theta-\theta_0) + Dr^2]\right\} \times r_0\,\mathrm{d}r_0\,\mathrm{d}\theta_0 - \int_0^{2\pi}\int_0^{\infty} A_{\mathrm{pb}}(r_0)E(r_0,\theta_0,0) \times\exp\left\{-\frac{\mathrm{i}k}{2B}[Ar_0^2 - 2rr_0\cos(\theta-\theta_0) + Dr^2]\right\} \times r_0\,\mathrm{d}r_0\,\mathrm{d}\theta_0\right\}. \tag{3}$$

Generally, we can expand the hard aperture function into a finite sum of complex Gaussian functions:

$$A_{\mathrm{pa}}(r_0) = \sum_{h=1}^{N} A_h \exp\left(-\frac{B_h}{a^2}r_0^2\right),$$
$$A_{\mathrm{pb}}(r_0) = \sum_{g=1}^{N} A_g \exp\left(-\frac{B_g}{b^2}r_0^2\right), \tag{4}$$

where $A_{h,g}$ and $B_{h,g}$ denote the expansion and Gaussian coefficients, respectively, which could be obtained by optimization–computation directly [13]. However, it should be noted that Eq. (4) is only an approximate expression for the hard aperture function. It has been shown that the larger the N is and the higher the simulation efficiency is [11,13–15].

2.1. Bessel–Gaussian beams

In the cylindrical coordinate system (r, θ, z) the field distribution $E(r, \theta.z)$ of Bessel–Gaussian beams at the plane $z = 0$ is given by Gori and Potvliege [1] and Belafhal [4]

$$E(r_0,\theta_0,0) = C_0 J_m(\alpha r_0)\exp\left(-\frac{r_0^2}{w_0^2}\right)\exp(-\mathrm{i}m\theta_0) \tag{5}$$

$$\text{with } \alpha = k\sin\phi, \tag{6}$$

where J_m is the mth-order Bessel function of the first kind and the parameters ϕ, C_0 and w_0 are the cone angle of the ideal non-apodized Bessel field (in the paraxial approximation), the amplitude at the origin and the spot size of the fundamental Gaussian mode. Fig. 1(a) shows the intensity distribution, normalized to unity at $r = 0$, for a mth-order Bessel–Gaussian beam with $m = 0$, 1, 2 at the $z = 0$ plane. The beam parameters are $\alpha = 24.81\,\mathrm{mm}^{-1}$ and $w_0 = 0.5\,\mathrm{mm}$.

Substituting from Eqs. (4) and (5) into Eq. (3) and recalling the integral formulae [16]

$$\int_0^{2\pi}\exp\left[\frac{\mathrm{i}k}{B}rr_0\cos(\theta-\theta_0)\right]\exp(-\mathrm{i}m\theta_0)\mathrm{d}\,\theta_0 = \mathrm{i}^m 2\pi J_m\left(\frac{k}{B}rr_0\right)\exp(-\mathrm{i}m\theta), \tag{7}$$