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A new strain energy function for modelling ligaments and tendons whose fascicles have a helical arrangement of fibrils

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ABSTRACT

A new strain energy function for the hyperelastic modelling of ligaments and tendons whose fascicles have a helical arrangement of fibrils is derived. The stress–strain response of a single fascicle whose fibrils exhibit varying levels of crimp throughout its radius is calculated and used to determine the form of the strain energy function. The new constitutive law is used to model uniaxial extension test data for human patellar tendon and is shown to provide an excellent fit, with the average relative error being 9.8%. It is then used to model shear and predicts that the stresses required to shear a tendon are much smaller than those required to uniaxially stretch it to the same strain level. Finally, the strain energy function is used to model ligaments and tendons whose *fascicles* are helical, and the relative effects of the fibril helix angle, the fascicle helix angle and the fibril crimp variable are compared. It is shown that they all have a significant effect; the fibril crimp variable governs the non-linearity of the stress–strain curve, whereas the helix angles primarily affect its stiffness. Smaller values of the helix angles lead to stiffer tendons; therefore, the model predicts that one would expect to see fewer helical sub-structures in stiff positional tendons, and more in those that are required to be more flexible.

1. Introduction

Ligaments and tendons are important connective tissues; ligaments connect bone to bone, providing stability and allowing joints to function correctly, and tendons connect muscle to bone to transfer forces generated by muscles to the skeleton. They both have a hierarchical structure consisting of several fibrous subunits (Kastelic et al., 1978; Screen et al., 2004), which, from largest to smallest, can be defined as follows: fascicles (50–400 μm diameter), fibrils (50–500 nm), sub-fibrils (10–20 nm), microfibrils (3–5 nm), and finally, the tropocollagen molecule (~1.5 nm). The geometrical arrangement of many of these subunits varies between different ligaments and tendons; for example, the patellar tendon's fascicles are coaligned with its longitudinal axis, whereas the anterior cruciate ligament's are helical (Shearer et al., 2014). The fibrils within a fascicle may also either be coaligned or helical with respect to its longitudinal axis (Yahia and Drouin, 1989). In both cases, the fibrils exhibit an additional waviness, called crimp, which is superimposed upon their average direction and varies in magnitude throughout the fascicle's radius (Kastelic et al., 1978; Yahia and Drouin, 1989). This intricate structure produces complex mechanical behaviour such as anisotropy, viscoelasticity and non-linearity, which varies between different ligaments and tendons (Benedict et al., 1968; Tipton et al., 1986). It is not currently known, however, which levels of the hierarchy are most influential in governing their mechanical performance.

To begin understanding these mechanical features, it is of interest to model their *elastic* properties, neglecting viscoelasticity. Elastic models are expected to be valid in both the low and extremely high strain rate limits where hysteresis is minimised. Ligament and tendon stress–strain behaviour under uniaxial tension is characterised by an initial non-linear region of increasing stiffness, termed the *toe-region*, followed by a linear region before the onset of failure (Fig. 1). Several authors have derived expressions to describe this behaviour (Frisen et al., 1969; Kastelic et al., 1980; Kwan and Woo, 1989); however, to consider more complex deformations, it is useful to characterise the elasticity of a material in terms of a strain energy function (SEF).

Many non-linear elastic SEFs have been proposed for soft tissues (Fung, 1967; Gou, 1970; Holzapfel et al., 2000), but few have focused specifically on ligaments and tendons. Whilst many SEFs are general enough to be applied to modelling ligaments and tendons, the majority of them contain variables that cannot be directly experimentally measured (Shearer, 2015). This limits their ability to analyse which physical quantities are most important in governing a specific ligament's or tendon's behaviour. Microstuctural models are better equipped to facilitate this analysis, provided their parameters can be experimentally determined.

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Nomenclature

$\theta_p(\rho)$	fibril crimp angle distribution
p	crimp angle distribution parameter
ρ	non-dimensional radial variable in fascicle
θ_o	crimp angle of outermost fibrils
$\mathbf{P}$, $\mathbf{p}$	unit vectors in fibril direction before/after fascicle stretch
α, ψ	fibril/fascicle helix angle
λ, ϵ	given fascicle stretch/strain
Λ	component of λ in fibril direction
$\Lambda_p(\rho)$	stretch in fibril direction as fibrils at radius ρ become taut
Λ^*, λ^*	stretch in fibril/fascicle direction that tautens outer fibrils
ϵ^*	critical fascicle strain as outer fibrils become taut
R_p	radius within which all fibrils are taut for a given λ
P_p	tensile load experienced by fascicle
$\sigma_p(\rho)$	contribution of fibril stress at radius ρ in fascicle direction
$\sigma_p^f(\rho)$ $\epsilon_p^f(\rho)$	stress/strain in fibril at radius ρ
E	fibril Young's modulus
τ_p	average traction in fascicle direction
β	$2(1-\cos^3\theta_o)/(3\sin^2\theta_o)$
W	strain energy function
I_1, I_4	isotropic/anisotropic strain invariant
ϕ	collagen volume fraction
W_m, W_f	component of W associated with matrix/fibrils
$\mathbf{B}$, $\mathbf{C}$	left/right Cauchy–Green tensor
$\mathbf{M}$, $\mathbf{m}$	undeformed/deformed fascicle direction vector
$\mathbf{F}$	deformation gradient
$\mathbf{T}$	Cauchy stress
Q	Lagrange multiplier
$\mathbf{T}_f$, $\mathbf{t}_f$	component of stress/traction associated with fascicles
$\hat{\mathbf{m}}$	unit vector in direction of $\mathbf{m}$
γ, η	constants defined in Eqs. (32) and (33)
μ	ground state shear modulus of ligament/tendon matrix
$\hat{\eta}$	constant defined in Eq. (41)
R, Θ, Z	circular cylindrical coordinates in undeformed configuration
A, a	undeformed/deformed tendon radius
L, l	undeformed/deformed tendon length
r, θ, z	circular cylindrical coordinates in deformed configuration
ζ	stretch in longitudinal direction of tendon
$\mathbf{e}_i$, $\mathbf{E}_j$	basis vectors in deformed/undeformed configuration
$\mathbf{n}$	outer unit normal to curved surface of tendon
S_{zz}, $S_{zz}^{\exp}$	theoretical/experimental longitudinal nominal stress
e	engineering strain
m	(machine precision)/2
δ, Δ	relative/absolute error
$\bar{\delta}$, $\bar{\Delta}$	average relative/absolute error
$\delta_{\max}$, $\Delta_{\max}$	maximum relative/absolute error
x, y, z	Cartesian coordinates in deformed configuration
X, Y, Z	Cartesian coordinates in undeformed configuration
γ_1, γ_2	shear strains
T_1, T_2	shear stresses
χ	function defined in Eq. (64)
C	constant of integration defined in Eq. (68)
N	resultant axial load acting on tendon
S	average force per unit undeformed area acting on tendon

Two examples of microstructural SEFs are those derived by Grytz and Meschke (2009) and Shearer (2015). Both are based on the geometrical arrangement of the fibrils within the fascicle and neglect any subunits below the fibril level (Fig. 2). Shearer considered a fascicle whose fibrils are coaligned with its axis, but have a distribution of crimp levels throughout its radius (Fig. 3(1) and (2)), whereas Grytz and Meschke considered a helical arrangement of fibrils, but neglected their crimp (Fig. 3(3) and (4)). Grytz and Meschke defined the angle that these fibrils make with the fascicle's longitudinal axis as the crimp angle, but this is not the usual definition of crimp. Here, this quantity is referred to as the *fibril helix angle*. The logical extension of these models is to allow the fibrils to be helically arranged *and* crimped (Fig. 3(5)); this is the case considered here. A scanning electron microscope (SEM) image displaying fibrils that are both helically arranged *and* crimped appears in Fig. 9 of Yahia and Drouin (1989).

A considerable amount of work has been dedicated to modelling other types of fibre-reinforced composite materials. Crossley et al. (2003), for example, derived analytical solutions that govern the bending and flexure of helically reinforced, anisotropic, linear elastic cylinders. This is built on a large body of literature on modelling cables (Cardou and Jolicouer, 1997) and rope (Costello, 1978, 1997). Adkins and Rivlin (1955) discussed finite deformations of materials that are reinforced by inextensible cords, and Spencer and Soldatos (2007) considered finite deformations of fibre-reinforced elastic solids whose fibres are capable of resisting bending. Whilst these general theories are extremely valuable for certain problems, to model the behaviour of a material with a microstructure as complex as a ligament or tendon requires a more specific model.

In this paper, a new SEF that governs the behaviour of a ligament or tendon with the microstructure described above is derived. In Section 2, the stress–strain response of a single fascicle is calculated and this relationship is used to determine the form of the new SEF in Section 3. The SEF is used to model the mechanical response of human patellar tendon to uniaxial extension and shear in Section 4. In Section 5, the case of a ligament or tendon with helical *fascicles* is explored and the relative effects of the *fibril crimp angle*, *fibril helix angle* and *fascicle helix angle* are analysed. Finally, the implications of the model are discussed in Section 6.

2. The stress–strain response of a fascicle with helically aligned fibrils

Kastelic et al. (1980) derived the stress–strain response of a fascicle with fibrils that are coaligned with its longitudinal axis, and Shearer (2015) adapted their method to derive analytical expressions for these relationships for different fibril crimp angle distributions. Here, this work is extended to ligaments and tendons whose fascicles have a helical arrangement of fibrils.

Kastelic et al. (1978) observed that crimp angle varies throughout the radius of a fascicle with longitudinal fibrils. It was then noted by Yahia and Drouin (1989) that this is also the case in fascicles with a helical arrangement of fibrils. The minimum crimp angle occurs at the fascicle's centre, the maximum at its edge. Therefore, assuming that only fully extended fibrils contribute to