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Unified viscoelasticity: Applying discrete element models to soft tissues with two characteristic times

Afshin Anssari-Benam [a,*], Andrea Bucchi [a], Dan L. Bader [b]

[a] School of Engineering, University of Portsmouth, Anglesea Road, Portsmouth PO1 3DJ, United Kingdom
[b] Faculty of Health Sciences, University of Southampton, Highfield, Southampton SO17 1BJ, United Kingdom

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ABSTRACT

Discrete element models have often been the primary tool in investigating and characterising the viscoelastic behaviour of soft tissues. However, studies have employed varied configurations of these models, based on the choice of the number of elements and the utilised formation, for different subject tissues. This approach has yielded a diverse array of viscoelastic models in the literature, each seemingly resulting in different descriptions of viscoelastic constitutive behaviour and/or stress-relaxation and creep functions. Moreover, most studies do not apply a single discrete element model to characterise both stress-relaxation and creep behaviours of tissues. The underlying assumption for this disparity is the implicit perception that the viscoelasticity of soft tissues cannot be described by a universal behaviour or law, resulting in the lack of a unified approach in the literature based on discrete element representations. This paper derives the constitutive equation for different viscoelastic models applicable to soft tissues with two characteristic times. It demonstrates that all possible configurations exhibit a unified and universal behaviour, captured by a single constitutive relationship between stress, strain and time as: $\sigma + A\dot{\sigma} + B\ddot{\sigma} = P\dot{\varepsilon} + Q\ddot{\varepsilon}$. The ensuing stress-relaxation $G(t)$ and creep $J(t)$ functions are also unified and universal, derived as $G(t) = c_1 e^{\frac{-A+\sqrt{A^2-4B}}{2B}t} + (\sigma_0 - c_1)e^{\frac{-A-\sqrt{A^2-4B}}{2B}t}$ and $J(t) = c_2 + (\varepsilon_0 - c_2)e^{-\frac{P}{Q}t} + \frac{\sigma_0}{P}t$, respectively. Application of these relationships to experimental data is illustrated for various tissues including the aortic valve, ligament and cerebral artery. The unified model presented in this paper may be applied to all tissues with two characteristic times, obviating the need for employing varied configurations of discrete element models in preliminary investigation of the viscoelastic behaviour of soft tissues.

1. Introduction

Biological tissues are known to exhibit complex constitutive behaviour, in which stress may depend on both strain and strain rate, as well as strain history (Fung, 1993; Bischoff et al., 2004). These characteristics suggest a behaviour that combines the properties of elastic solids and viscous fluids, and therefore biological tissues are generally known to respond in a viscoelastic manner to mechanical perturbations (Jamison et al., 1968). From a biomechanics point of view, the difference between the viscoelastic response of a tissue, and purely elastic or viscous responses, lies essentially in the relationship between stress, strain and time.

To address such relationships in soft tissues, appropriate models that describe a mathematical representation of the phenomenon of viscoelasticity are required. While some studies suggest that aspects of viscoelastic behaviour of tissues, such as stress-relaxation and creep, may be initiated at the extracellular-matrix level, e.g. by the viscous fluid-like behaviour and characteristics of glycosaminoglycans (GAGs) (Ratcliffe and Mow, 1996; Anssari-Benam et al., 2011a, 2011b) and fibre-sliding (Gupta et al., 2010; Screen et al., 2013), rheological models have traditionally been the popular choice to characterise the viscoelastic behaviour and properties of soft tissues.

In general, rheological models correspond to either continuous spectral, or discrete element, mathematical representations (Jamison et al., 1968; Fung, 1993; Li and Xu, 2006). While the physical demonstration of both models is manifested by configurations of a finite number of springs and dashpots, the corresponding mathematical expressions are addressed differently. Continuous spectral representations are obtained by solving an integral equation for the relaxation spectrum, under the assumption that the applied strain and the resulting stress response are known (Jamison et al., 1968). Perhaps the most celebrated type of

* Corresponding author. Tel.: +44 23 9284 2187; fax: +44 23 9284 2351.
E-mail address: afshin.anssari-benam@port.ac.uk (A. Anssari-Benam).

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these models utilised in biomechanical studies is the quasi-linear viscoelasticity (QLV) model, successfully applied to various tissues ranging from cardiac muscles (Pinto and Patitucci, 1980) to ligaments (Woo et al., 1993) and tendons (Sarver et al., 2003). The governing equations for discrete element models are derived based on the local stress/strain/strain rate equations of the incorporated spring and dashpot elements, and how they are related to each other and with the global stress–strain relationships (Jamison et al., 1968). Examples of application of discrete element models to soft tissues include characterisation of the rate effects (Anssari-Benam et al., 2011a) and stress-relaxation and creep behaviour of heart valves (Liao et al. 2007; Anssari-Benam et al., 2011b), tendons (Hooley and Cohen, 1979) and arteries (Rehal et al., 2006).

While the application of discrete element models to soft tissues have mainly been made within the context of linear viscoelasticity, which would undoubtedly introduce approximations and simplifications to the analysis (for example see Provenzano et al. (2002) and Anssari-Benam (2014)), discrete element models have been reported to provide good agreement with the experimental data. The preliminary experiments in characterising the viscoelasticity of soft tissues incorporate quasi-static loading regimes often in the form of stress-relaxation and creep tests, as well as tensile loading under various deformation rates. Discrete element viscoelastic models facilitate the quantification of the respective stress-relaxation and creep moduli, characteristic times, and the rate effects, in a mathematically and conceptually easy way, by incorporating combination of spring and dashpot elements. Additionally, different trends of experimental data may be fitted to these models relatively easily, by altering the number of the elements in the model or the model configuration. These attributes have made discrete element viscoelastic models a popular choice in investigating and characterising the viscoelasticity of soft tissues.

However, this apparent freedom in employing various element numbers or configurations has rendered a diverse array of viscoelastic models in the literature, each seemingly resulting in different descriptions of viscoelastic constitutive behaviour. This diversity, in turn, has given rise to a perception that the constitutive viscoelastic relationship between stress, strain and time in soft tissues may not be universal when characterised using discrete element models, but rather may depend on the employed number and configuration of the elements. Such disparity has made direct comparisons between the quantified viscoelastic properties of different tissues highly problematic, as the differences in values may be partly attributed to the application of 'different' viscoelastic models. This problem becomes more pronounced when comparing the reported viscoelastic properties of a specific soft tissue, where different configurations have been employed. For example, different relaxation times for aortic valve tissue have been reported when characterised by QLV (Sauren et al., 1983) which is a Kelvin-based model, Maxwell-type exponential decay (Lee and Vesely, 1995) and Prony series (Anssari-Benam et al., 2011b). A similar diversity of parametric values has been identified in the literature for arteries, tendons and other soft tissues.

Due care must therefore be observed in choosing adequate number and appropriate configuration of elements that could suitably describe the experimentally observed viscoelastic behaviour of soft tissues. Previous studies have established that two characteristic time scales, referred to as "fast" and "slow" times or "short-" and "long-" time memory, are sufficient to capture and characterise the time-dependent behaviour of many soft tissues (Fung, 1993; Pioletti and Rakotomanana, 2000; Banks et al., 2011). The fast and slow characteristic times are the macroscopic time scales that are required for a tissue to return to its equilibrium state, after exposure to external mechanical perturbations. The fast characteristic time implies that a short time is required for the tissue to retain the equilibrium state, while a slow characteristic time reflects a long time-scale for the tissue to return to its original reference. Soft tissues possess both short and long characteristic time scales, presenting a fast initial recovery followed by a much slower equilibrium kinematics, in stress-relaxation tests. In discrete element modelling, this two-characteristic time behaviour can be represented by two dashpots, the mechanical elements introducing the time/rate effects, together with two spring elements, in an arrangement such that similar elements would not form parallel or series configurations. In addition, most soft tissues exhibit both stress-relaxation and creep behaviours when subjected to the respective loading conditions. An appropriate viscoelastic model therefore must also be capable of characterising both those behaviours under those conditions.

These critical axioms, however, have often been overlooked in the discrete element models developed in the literature. Indeed, literature suggests a variety of Kelvin-based, Maxwell-based, or standard linear solid type discrete element models that have been used for different tissues, or even for the same tissue, with various element numbers, only suitable for characterising a particular viscoelastic behaviour. For example, the standard linear solid model while successful in addressing force-displacement relationships in some biological entities (e.g. axonal microtubules by Shamloo et al. (2015)), can only render a single characteristic relaxation time and as such may not be applicable to tissues with two characteristic times. Similarly, Maxwell-based models can only accurately characterise stress-relaxation, and Kelvin-based models can only accurately characterise creep behaviour of soft tissues. Studies have therefore often favoured the application of one model to describe the stress-relaxation and a separate model to describe the creep behaviour, even for the same tissue (Hooley and Cohen, 1979; Thornton et al., 1997; Anssari-Benam et al., 2011b). The biomechanics literature has therefore not adequately addressed the important question of whether the viscoelasticity of soft tissues with two characteristic times is a universal behaviour, and has subsequently not provided a unified discrete element model with single constitutive relationship between stress, strain and time, or stress relaxation and creep functions applicable to all such tissues.

In this paper, the constitutive equation for discrete element viscoelastic models applicable to soft tissues with two characteristic times, i.e. four-element representations, are derived and presented. It is shown that all possible configurations of these models exhibit a universal behaviour, with the following unified relationship between stress, strain and time:

$$\begin{cases} \sigma + A\dot{\sigma} + B\ddot{\sigma} = P\dot{\varepsilon} + Q\ddot{\varepsilon} & \text{(a)} \\ G(t) = c_1 e^{\frac{-A+\sqrt{A^2-4B}}{2B}t} + (\sigma_0 - c_1)e^{\frac{-A-\sqrt{A^2-4B}}{2B}t} & \text{(b)} \\ J(t) = c_2 + (\varepsilon_0 - c_2)e^{-\frac{P}{Q}t} + \frac{\sigma_0}{P}t & \text{(c)} \end{cases} \quad (1)$$

where σ denotes stress, ε is strain, $G(t)$ and $J(t)$ are the resulting stress relaxation and creep functions, respectively, and A, B, P and Q are constants determined from elastic and viscous damping moduli of the elements in the model, as will be shown in the next section. Our analysis concludes that different four-element viscoelastic models applicable to soft tissues with two characteristic times, all lead to a universal response, characterised by a single mathematical representation given in Eq. (1), with single universal stress-relaxation and creep functions. Application of these functions to experimental data obtained from stress-relaxation and creep tests reported in the literature for a range of different types of tissues including aortic valve, ligament and cerebral artery is also presented.

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