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Quantification of wall shear stress using a finite-element method in multidimensional phase-contrast MR data of the thoracic aorta

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ABSTRACT

We present a computational method for calculating the distribution of wall shear stress (WSS) in the aorta based on a velocity field obtained from two-dimensional (2D) phase-contrast magnetic resonance imaging (PC-MRI) data and a finite-element method. The WSS vector was obtained from a global least-squares stress-projection method. The method was benchmarked against the Womersley model, and the robustness was assessed by changing resolution, noise, and positioning of the vessel wall. To showcase the applicability of the method, we report the axial, circumferential and magnitude of the WSS using in-vivo data from five volunteers. Our results showed that WSS values obtained with our method were in good agreement with those obtained from the Womersley model. The results for the WSS contour means showed a systematic but decreasing bias when the pixel size was reduced. The proposed method proved to be robust to changes in noise level, and an incorrect position of the vessel wall showed large errors when the pixel size was decreased. In volunteers, the results obtained were in good agreement with those found in the literature. In summary, we have proposed a novel image-based computational method for the estimation of WSS on vessel sections with arbitrary cross-section geometry that is robust in the presence of noise and boundary misplacements.

1. Introduction

Two-dimensional cine phase-contrast magnetic resonance imaging (2D CINE PC-MRI) and three-dimensional (3D) CINE PC-MRI have been used non-invasively to obtain qualitative and quantitative information on the cardiovascular system. Several numerical procedures have recently been proposed to evaluate flow patterns, determine the wall shear stress (WSS) distribution, and calculate pressure difference maps (Oshinski et al., 1995; Tyszka et al., 2000; Ebbers et al., 2001; Barker et al., 2010; Bock et al., 2010, 2011). These methods have shown the potential of 2D and 3D CINE PC-MRI for assessing different cardiovascular diseases (Wigström et al., 1999; Weigang et al., 2008; Boussel et al., 2009; Kafka and Mohiaddin, 2009; Markl et al., 2010; Cecchi et al., 2011; Frydrychowicz et al., 2011; Francois et al., 2012). In particular, the evaluation of the WSS distribution in the aortas of healthy volunteers (Stalder et al., 2008; Frydrychowicz et al., 2009a) and patients has recently been reported by several groups (Frydrychowicz et al., 2009b; Barker et al., 2010; Harloff et al., 2010; Bieging et al., 2011). It has been shown that different WSS-related parameters – including the axial and circumferential components of WSS and the oscillatory shear index – have potential for assessing vascular function in several cardiovascular diseases such as atherosclerosis, aneurysms, stenosis and restenosis (Cecchi et al., 2011).

The estimation of the WSS distribution from 2D CINE PC-MRI goes back to the work of Oshinski et al. (1995), Morgan et al. (1998a,b) and Oyre et al. (1998). In Oshinski et al. the WSS is calculated from the product of the fluid viscosity and the velocity gradient at the wall, correcting for the wall position using MR data.

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Assuming an axial flow profile in a perfectly cylindrical vessel (Poiseuille flow model), Oyre et al. fitted a paraboloid to the through-plane velocity profile measured at a boundary layer (Oyre et al., 1998). From the fit, they were able to compute the axial velocity gradient, and in turn the WSS in the carotid arteries of seven healthy volunteers. Whereas this approach may be valid for small and rounded vessels, it cannot capture other flow patterns commonly found in larger vessels, where the velocity profile does not follow a parabolic distribution. Morgan et al. followed a different approach in which the tangential, radial and axial velocity gradients were numerically estimated using a finite-difference scheme to compute the WSS tensor at the left and right pulmonary arteries (Morgan et al., 1998a,b). However, it is well known that the finite-difference method cannot effectively handle complex geometries such as those found in the cardiovascular system, neither can it impose boundary conditions on irregular surfaces in a direct manner (Zienkiewicz et al., 2005). To account for arbitrary cross-section shapes, Stalder et al. (2008) used cubic B-spline interpolations to smoothly describe the lumen contours as well as to obtain a continuous and smooth description of the velocity field. This method has been the standard for quantifying the WSS from 2D cine PC-MRI. However, as noted by Stalder et al., the computation of WSS using B-spline interpolations on 2D CINE PC-MRI and on reformatted slices from 3D CINE PC-MRI, both with 3D velocity encoding, introduces important approximation errors due to limited spatial resolution and numerical differentiation of the velocity field, which in the case of a Poiseuille flow can be as high as 40% in the estimation of WSS.

In this work we propose a finite-element-based methodology to compute the WSS at arbitrary plane sections of the thoracic aorta from a velocity field given by 2D CINE PC-MRI data. The finite-element method has a well-established reputation for efficiently representing 3D complex geometries, and has been successfully employed in patient-specific cardiovascular and cardiac simulations (Taylor and Figueroa, 2009; Xiao et al., 2013; Hurtado and Kuhl, 2014), providing in general a robust means for cardiovascular modeling and computation, with numerical convergence that can be rigorously proven (Hurtado and Henao, 2014). In particular, finite-element methods have been used in computational fluid dynamic (CFD) simulations to obtain different hemodynamic parameters on the basis of 3D models build from angiography images and boundary conditions from 2D PC-MRI (LaDisa et al., 2011; Goubergrits et al., 2014). Nevertheless, as far as we are aware, finite elements have not been applied directly to process the velocity data from 2D PC-MRI and in turn to obtain the WSS.

To estimate the velocity gradients, the domain of interest is discretized using triangular elements, and the velocities at the center of each voxel are interpolated using a conforming finite-element approximation of the velocity field. In order to improve the accuracy of the computed strain and stress fields, several a posteriori stress-recovery methods have been proposed in the literature (Zienkiewicz et al., 2005). Here, we adopt a global least-squares stress projection method (Oden and Brauchli, 1971), which has been shown to be super-convergent for linear elements (Zienkiewicz and Zhu, 1992), exhibiting in some cases a better performance than alternative methods (Heimsund et al., 2002). We tested the proposed methodology using a Womersley flow profile as a benchmark. The robustness of the method was assessed under different levels of resolution and noise and incorrect positioning of the vessel wall. To showcase the applicability of the method, we report the axial, circumferential and magnitude of the WSS using in-vivo data.

2. Theory

2.1. Computation of the wall shear stress using a finite-element method

The shear stress vector and magnitude at the vessel wall were computed using the procedure described in the electronic supplementary material (see Appendix: finite element formulation), which we briefly summarize next. The velocity field was obtained at a discrete set of pixels using 2D CINE PC-MRI. Using linear triangular finite-element interpolations, the velocity-component field $u_i^{FEM}(\boldsymbol{x},t)$ is continuously described by the expression:

$$u_i^{FEM}(\boldsymbol{x},t) = \sum_{A\in\eta} N_A(\boldsymbol{x}) v_{iA}(t), \tag{1}$$

where $N_A(\boldsymbol{x})$ is the finite-element shape function associated to node A, $v_{iA}(t)$ is the i-th velocity component at the node A at time t, and η is the set of all nodes of the triangular mesh used as discretization of the section under study. Based on Eq. (1), the shear stress tensor components can be approximated using a global least-squares stress projection method (Oden and Brauchli, 1971; Hinton and Campbell, 1974), which consists in approximating the stress field $\tau^S(\boldsymbol{x},t)$ by:

$$\tau^S(\boldsymbol{x},t) = \sum_{A\in\eta} N_A(\boldsymbol{x})\, \tau_A(t), \tag{2}$$

where τ_A is the nodal smoothed value of the stress components obtained from a global least-squares minimization of the stress L2 error. Once all the shear-component fields are obtained, the shear stress tensor $\boldsymbol{\tau}$ at any point in the domain of interest, and particularly at the domain boundaries (i.e. vessel wall), can be estimated using Eq. (2). Let $\vec{\boldsymbol{n}}$ be the inward unit vector normal to the vessel wall at a particular point of interest. Then, the WSS vector corresponding to the shear stress tensor takes the form:

$$\vec{\boldsymbol{t}} = \boldsymbol{\tau}\cdot\vec{\boldsymbol{n}}. \tag{3}$$

For the purpose of this work, we consider the axial, circumferential and magnitude of the WSS vector projected over the lumen contour $\vec{t}_{proy}$:

$$\vec{t}_{proy} = \vec{n} \times \left(\vec{\boldsymbol{t}} \times \vec{n}\right). \tag{4}$$

From $\vec{t}_{proy} = [t_X, t_Y, t_Z]$, the axial component, t_Z, represented the projection in the longitudinal direction, and the circumferential component represented the projection along the lumen circumference, which was calculated as $\sqrt{t_x^2 + t_z^2}$.

The method just described was implemented in Python language. It is important to mention that we defined the velocities in the boundary of our mesh to be equal to zero, following a no-slip boundary assumption.

3. Methods

3.1. Womersley flow model and robustness analysis

To evaluate the stability and robustness of the method, we generated synthetic velocity profiles using the Womersley model (Eq. (5)) (Womersley, 1955). A detailed explication of the Womersley model is given in electronic supplementary material (Womersley formulation – see Appendix). From the Womersley model the velocity inside a cylinder is given by:

$$vel(x,y,t) = \frac{A_0}{4\mu}\cdot\left(R^2 - r(x,y)^2\right) + Re\left[PG_m(t)\cdot\frac{1}{i\rho\omega}\cdot\left\{1 - \frac{J_0\left(r(x,y)\sqrt{\frac{\omega}{\nu}}\, i^{3/2}\right)}{J_0\left(R\sqrt{\frac{\omega}{\nu}}\, i^{3/2}\right)}\right\}\right], \tag{5}$$

where $vel(x,y,t)$ is the velocity in the point (x,y) (radius r) at time t, in the interior of a cylinder of length L and radius R; ρ is the blood density, μ is the viscosity