Journal of Biomechanics 46 (2013) 2761–2768

Contents lists available at ScienceDirect

Journal of Biomechanics

journal homepage: www.elsevier.com/locate/jbiomech
www.JBiomech.com

Finite element modeling of impulsive excitation and shear wave propagation in an incompressible, transversely isotropic medium

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ARTICLE INFO

Article history:
Accepted 5 September 2013

Keywords:
Finite element modeling
Impulsive excitation
Acoustic Radiation Force Impulse (ARFI)
Transversely isotropic material

ABSTRACT

Elastic properties of materials can be measured by observing shear wave propagation following localized, impulsive excitations and relating the propagation velocity to a model of the material. However, characterization of anisotropic materials is difficult because of the number of elasticity constants in the material model and the complex dependence of propagation velocity relative to the excitation axis, material symmetries, and propagation directions. In this study, we develop a model of wave propagation following impulsive excitation in an incompressible, transversely isotropic (TI) material such as muscle. Wave motion is described in terms of three propagation modes identified by their polarization relative to the material symmetry axis and propagation direction. Phase velocities for these propagation modes are expressed in terms of five elasticity constants needed to describe a general TI material, and also in terms of three constants after the application of two constraints that hold in the limit of an incompressible material. Group propagation velocities are derived from the phase velocities to describe the propagation of wave packets away from the excitation region following localized excitation. The theoretical model is compared to the results of finite element (FE) simulations performed using a nearly incompressible material model with the five elasticity constants chosen to preserve the essential properties of the material in the incompressible limit. Propagation velocities calculated from the FE displacement data show complex structure that agrees quantitatively with the theoretical model and demonstrates the possibility of measuring all three elasticity constants needed to characterize an incompressible, TI material.

1. Introduction

Physicians use manual palpation as part of the diagnostic process because diseased tissue is often stiffer than surrounding healthy tissue. Recently, a number of imaging methods have been developed for characterizing tissue stiffness *in vivo* by exciting shear waves in the tissue and measuring the shear wave propagation speed (Sarvazyan et al., 1998; Sandrin et al., 2003). These systems generate shear waves in tissue using either external mechanical excitation coupled to the body wall (Yin et al., 2007; Huwart et al., 2006) or acoustic radiation force impulse (ARFI) excitations to remotely palpate tissue at the focal region of an acoustic beam (Bercoff et al., 2004; Chen et al., 2004; Nightingale et al., 2003). Wave propagation is monitored in space and time by a real-time imaging modality such as magnetic resonance imaging or ultrasound tracking, and the tissue stiffness is determined quantitatively by measuring the shear wave propagation speed.

Two elasticity constants are required to characterize a linear, elastic, isotropic material (Lai et al., 1999). For example, the Lamé constants λ and μ could be specified and used to calculate related constants such as Young's modulus, bulk modulus, and Poisson's ratio. For nearly incompressible materials such as many biological tissues, λ and μ often differ by a factor on the order of 10^6, with a corresponding difference in longitudinal and shear wave speeds of 10^3. Finite element (FE) models of these materials use a Poisson ratio nearly equal to the limiting value of 0.5 which characterizes an incompressible material (Palmeri et al., 2005). Typically, ultrasonic or magnetic resonance imaging methods used to track wave propagation in these materials only attempt to measure the shear modulus.

The characterization of anisotropic materials requires more elasticity constants in the material model. For example, in a linear, elastic, transversely isotropic (TI) material, a symmetry axis exists and the material can be characterized by five elasticity constants (Lai et al., 1999). Muscle is an example of a TI material with the symmetry axis defined by the orientation of the muscle fibers. Measurements of shear wave speed for propagation along and across the muscle fibers have been reported (Gennisson

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http://dx.doi.org/10.1016/j.jbiomech.2013.09.008

et al., 2003; Papazoglou et al., 2006; Gennisson et al., 2010; Royer et al., 2011). Recently, Wang et al. (2013a) have measured the angular dependence of phase and group velocities for shear wave propagation in muscle using 3D volumetric measurements performed using a 2D matrix array (Wang et al., 2013b).

Wave propagation in anisotropic materials has been studied in several diverse areas. One example of wave propagation in TI materials occurs in layered media in seismology where the symmetry axis is perpendicular to the layers (Carcione, 2001; Tsvankin, 2001). Another example occurs in crystallography (Auld, 1990; Musgrave, 1970), where the hexagonal crystal structure has TI symmetry.

In this study, we consider FE modeling of impulsive excitation and wave propagation in an incompressible, TI medium. The focus of the study centers around three primary components as follows. First, the elasticity constants needed to describe an incompressible, TI material are identified, and FE models are constructed using a nearly incompressible material model which preserves the essential properties of the incompressible model. Second, the geometrical configuration for the excitation, material symmetry axis, and wave propagation are chosen to simulate the experimental setup commonly used in ARFI excitation and ultrasonic tracking experiments. Finally, the angular dependence of measured wave velocities is compared with theoretical predictions to demonstrate how the elasticity constants can be determined from experimental measurements. The results of the study demonstrate that with an appropriate experimental configuration, it is possible to measure all of the elasticity constants required to characterize an incompressible, TI material such as muscle.

2. Wave propagation in an incompressible, TI medium

2.1. TI materials

In the limit of small displacements, such as those produced in ARFI excitations, the stress–strain relationship in an anisotropic material is linear and can be described by a generalized Hooke's law as

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \tag{1}$$

where c_{ijkl} are the components of a fourth-order stiffness tensor and summation over repeated indices is implied. Symmetries of the stress and strain tensors and the existence of a strain energy allow the stiffness tensor to be expressed in terms of 21 independent elements (Lai et al., 1999). Then the stress–strain relations (1) can be written as a matrix product,

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} = C \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{pmatrix} \tag{2}$$

where C is a 6×6 symmetric stiffness matrix (Lai et al., 1999).

Transversely isotropic (TI) materials possess an axis of symmetry $\hat{A}$ such that reflection symmetry exists across every plane parallel to $\hat{A}$ and rotation symmetry exists about $\hat{A}$. Muscle is an example of a TI material with the axis $\hat{A}$ given by the direction of the muscle fibers. These symmetries imply that five independent elastic constants are required to specify the stiffness matrix C (Lai et al., 1999). Relative to an x_1, x_2, x_3 coordinate system oriented so that $\hat{x_3} = \hat{A}$, C is given by (Lai et al., 1999)

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{11} & C_{13} & & & \\ C_{13} & C_{13} & C_{33} & & & \\ & & & C_{55} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{pmatrix} \tag{3}$$

where missing elements are zero and

$$C_{66} = \frac{C_{11}-C_{12}}{2}. \tag{4}$$

For TI materials, the elements of the stiffness matrix can be expressed in terms of Young's moduli E_T and E_L, Poisson's ratios ν_{LT} and ν_{TT}, and shear moduli μ_T and μ_L where the longitudinal (L) and transverse (T) directions are defined relative to the material symmetry axis. In terms of the compliance matrix $S = C^{-1}$ (Lai et al., 1999),

$$S = C^{-1} = \begin{pmatrix} 1/E_T & -\nu_{TT}/E_T & -\nu_{LT}/E_L & & & \\ -\nu_{TT}/E_T & 1/E_T & -\nu_{LT}/E_L & & & \\ -\nu_{LT}/E_L & -\nu_{LT}/E_L & 1/E_L & & & \\ & & & 1/\mu_L & & \\ & & & & 1/\mu_L & \\ & & & & & 1/\mu_T \end{pmatrix} \tag{5}$$

where

$$\mu_T = \frac{E_T}{2(1+\nu_{TT})}. \tag{6}$$

2.2. Incompressible TI materials

For an incompressible material, the fractional volume change, or dilatation e, of an infinitesimal volume subjected to stresses must be zero. The dilatation is given by the trace of the strain tensor (Lai et al., 1999) and can be expressed in terms of the elastic constants using (2) and (5) as

$$e = \frac{1}{E_T}\left(1-\nu_{TT}-\nu_{LT}\frac{E_T}{E_L}\right)(\sigma_{11}+\sigma_{22}) + \frac{1}{E_L}(1-2\nu_{LT})\sigma_{33}. \tag{7}$$

For an incompressible material, both of the terms on the right hand side of (7) must be zero, and the Poisson ratios satisfy two conditions,

$$\nu_{TT} = 1-\frac{E_T}{2E_L} \tag{8}$$

and

$$\nu_{LT} = \tfrac{1}{2}. \tag{9}$$

These results agree with those of Papazoglou et al. (2006). Thus, three independent elastic constants are needed to describe an incompressible TI material compared to five constants in the general TI case. In the following, results are expressed in terms of the constants μ_T, μ_L, and E_T/E_L.

2.3. Wave dynamics

Assuming no external body forces, the acceleration of an infinitesimal volume of material with density ρ can be expressed using Newton's second law in terms of the displacement $\vec{u}$ and the stress gradient across the volume as (Tsvankin, 2001)

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho\frac{\partial^2 u_i}{\partial t^2}. \tag{10}$$