Journal of Biomechanics 47 (2014) 3361–3372

Contents lists available at ScienceDirect

Journal of Biomechanics

journal homepage: www.elsevier.com/locate/jbiomech
www.JBiomech.com

Enhanced global mathematical model for studying cerebral venous blood flow

Lucas O. Müller *, Eleuterio F. Toro

Laboratory of Applied Mathematics, University of Trento, Via Mesiano 77, 38123 Trento, Italy

ARTICLE INFO

Article history:
Accepted 1 August 2014

Keywords:
Global circulation
Mathematical model
Cerebral blood flow
Intracranial pressure
Starling resistor

ABSTRACT

Here we extend the global, closed-loop, mathematical model for the cardiovascular system in Müller and Toro (2014) to account for fundamental mechanisms affecting cerebral venous haemodynamics: the interaction between intracranial pressure and cerebral vasculature and the Starling-resistor like behaviour of intracranial veins. Computational results are compared with flow measurements obtained from Magnetic Resonance Imaging (MRI), showing overall satisfactory agreement. The role played by each model component in shaping cerebral venous flow waveforms is investigated. Our results are discussed in light of current physiological concepts and model-driven considerations, indicating that the Starling-resistor like behaviour of intracranial veins at the point where they join dural sinuses is the leading mechanism. Moreover, we present preliminary results on the impact of neck vein strictures on cerebral venous hemodynamics. These results show that such anomalies cause a pressure increment in intracranial cerebral veins, even if the shielding effect of the Starling-resistor like behaviour of cerebral veins is taken into account.

1. Introduction

Recent developments on the potential link between extracranial venous anomalies and neurological conditions have increased interest on the physiology of cerebral venous return, currently, a poorly understood subject (Stoquart-ElSankari et al., 2009). A prominent example is Chronic CerebroSpinal Venous Insufficiency (CCSVI) and its potential link to Multiple Sclerosis (MS) (Zamboni et al., 2009). Interest in this area has been further increased by very recent findings on altered cerebrospinal fluid dynamics in MS patients (Magnano et al., 2012) and by reported improvements of such dynamics, as well the clinical course of the disease, after treatment of MS/CCSVI patients with percutaneous transluminal angioplasty (PTA) (Zivadinov et al., 2013).

In this paper we are concerned with the development of a mathematical model to study cerebral haemodynamics. To this end we construct an extension of the closed-loop model for the cardiovascular system reported in Müller and Toro (2014) to account for two relevant factors. First we deal with the interaction of the cerebral vasculature with the pulsating intracranial pressure, for which we adopt the model proposed in Ursino (1988) and Ursino and Lodi (1997); this model describes the variation of intracranial pressure in time in terms of the variation of cerebral blood volume. Second, we incorporate a model to account for the Starling-resistor like behaviour of cerebral veins. This is supported by experimental evidence that shows that pressure in cerebral veins is always higher than intracranial pressure, for a wide range of intracranial pressures, independently of the pressure in downstream vessels, such as the dural sinuses (Johnston et al., 1974; Luce et al., 1982). The underlying mechanism is still the subject of debate, with some researchers speaking in favour of a purely hydraulic mechanism (Anile et al., 2009) and others hypothesizing a control mechanism (Dagain et al., 2009). Moreover, there is evidence of distinct morphological and mechanical properties of the terminal portion of cerebral veins, in the vicinity of dural sinuses (Vignes et al., 2007; Dagain et al., 2008, 2009; Chen et al., 2012). In order to model this behaviour, we have added cerebral veins to the venous network in Müller and Toro (2014) and implemented a simple model for Starling-resistor elements, proposed in Mynard (2011). These elements are placed in the vicinity of the point where cerebral veins drain into the dural sinuses. Our model is partially validated by comparing our computational results with MRI-derived flow measurements, obtaining satisfactory agreement. Computational results are discussed in light of current knowledge of the physiology of cerebral venous haemodynamics. As an example, we include here some preliminary results on the impact of neck vein strictures on cerebral venous haemodynamics. These results show that such anomalies cause a

* Corresponding author.
E-mail address: lucas.mueller@gmail.com (L.O. Müller).

http://dx.doi.org/10.1016/j.jbiomech.2014.08.005
0021-9290/

pressure increase in intracranial cerebral veins. The proposed mathematical model constitutes a computational tool suitable for the study of pathologies related to extracranial venous anomalies and their interaction with intracranial haemodynamics. This will be the subject of future investigations.

The rest of the paper is structured as follows. Section 2 describes the extensions made to the model reported in Müller and Toro (2014). Section 3 contains computational results, comparison with MRI-derived measurements and a discussion of computational results. Section 4 contains an example to illustrate the applicability of the present model, while Section 5 includes a summary and conclusions.

2. Methods

The model presented in this paper is an extension of the closed-loop model for the cardiovascular system presented in Müller and Toro (2014) to which the reader is referred to for further details.

2.1. Mathematical model of the cardiovascular system

Our mathematical model includes a one-dimensional description of the networks of major arteries and veins, and lumped-parameter models for the heart, the pulmonary circulation and capillary beds linking arteries and veins, see Fig. 1. Geometrical information for major head and neck veins is obtained from segmentation of MRI data. This patient-specific characterization allows us to compare computational results *versus* patient-specific MRI-derived flow quantification data, see Müller and Toro (2014).

One-dimensional blood flow in elastic vessels is described by the following system of non-linear, first-order hyperbolic equations:

$$\begin{cases} \partial_t A + \partial_x q = 0, \\ \partial_t q + \partial_x \left(\hat{\alpha} \frac{q^2}{A} \right) + \frac{A}{\rho} \partial_x p = -f, \end{cases} \tag{1}$$

here x is the axial coordinate of the vessel; t is the time; $A(x,t)$ is the cross-sectional area of the vessel; $q(x,t)$ is the flow rate; $p(x,t)$ is the average internal pressure over a cross-section; $f(x,t)$ is the friction force per unit length of the tube; ρ is the fluid density and $\hat{\alpha}$ is a coefficient related to the assumed velocity profile. In what follows we assume ρ=constant and $\hat{\alpha}=1$ (blunt velocity profile). To close the system we adopt a tube law relating $p(x,t)$ to $A(x,t)$ and other parameters, namely

$$p(x,t) = p_e(x,t) + K(x)\left[\left(\frac{A(x,t)}{A_0(x)} \right)^m - \left(\frac{A(x,t)}{A_0(x)} \right)^n \right] + P_0, \tag{2}$$

here $p_e(x,t)$ is the external pressure. $K(x)$, m, n, $A_0(x)$ and P_0 are parameters that account for mechanical and geometrical properties of the vessel. For a discussion on the choice of these parameters for both arteries and veins, see Müller and Toro (2014) and references therein. Fig. 1 depicts a schematic representation of the global model. Fig. 2 shows the venous network for the head and neck used here. Cortical cerebral veins draining into the superior and inferior sagittal sinuses have been added to the venous network in Müller and Toro (2014). Note that there is high variability in the number of veins draining into the superior sagittal sinus; our choice is in line with the numbers reported in Vignes et al. (2007). Moreover, a Starling-resistor element was added to the end of each cerebral vein (see Section 2.3). Table 1 shows geometrical and mechanical properties of vessels added or changed with respect to the venous network of Müller and Toro (2014). As a consequence of the venous network extension, the lumped-parameter compartment linking middle and anterior cerebral arteries to the inferior and superior sagittal sinuses has also been modified. The new configuration of this lumped compartment is shown in Fig. 3 and coefficients for lumped elements are found in Table 2.

2.2. Intracranial pressure

The cranial cavity is conventionally regarded as a space of fixed volume containing the brain parenchyma, the cerebrospinal fluid (CSF) and the cerebral vasculature. The cranial cavity is then connected to the spinal cavity, which exhibits elastic behaviour, allowing for volume changes. Variations in intracranial blood volume produce fluctuations of intracranial pressure and, consequently, exchange of CSF between the intracranial and spinal subarachnoid spaces. For intracranial pressure p_{ic}, here we adopt the model proposed in Ursino (1988) and Ursino and Lodi (1997) given by

$$C_{ic} \frac{dp_{ic}}{dt} = \frac{dV_{cv}}{dt} + \frac{p_c - p_{ic}}{R_f} - \frac{p_{ic} - p_{sss}}{R_0}, \tag{3}$$

here p_c and p_{sss} are capillary and superior sagittal sinus pressures, respectively. V_{cv} is the volume of the cerebral vasculature, given by the sum of the volume occupied by arteries, arterioles, capillaries, venules and veins inside the cranium. C_{ic} is the intracranial compliance, given as

$$C_{ic} = \frac{1}{k_e p_{ic}}, \tag{4}$$

where k_e is the elastance coefficient of the craniospinal system. Here we use $k_e = 0.15\ \mathrm{ml}^{-1}$ (Ursino, 1988). R_f and R_0 are CSF filtration and re-absorption resistances. CSF filtration from the subarachnoid space towards the dural sinuses

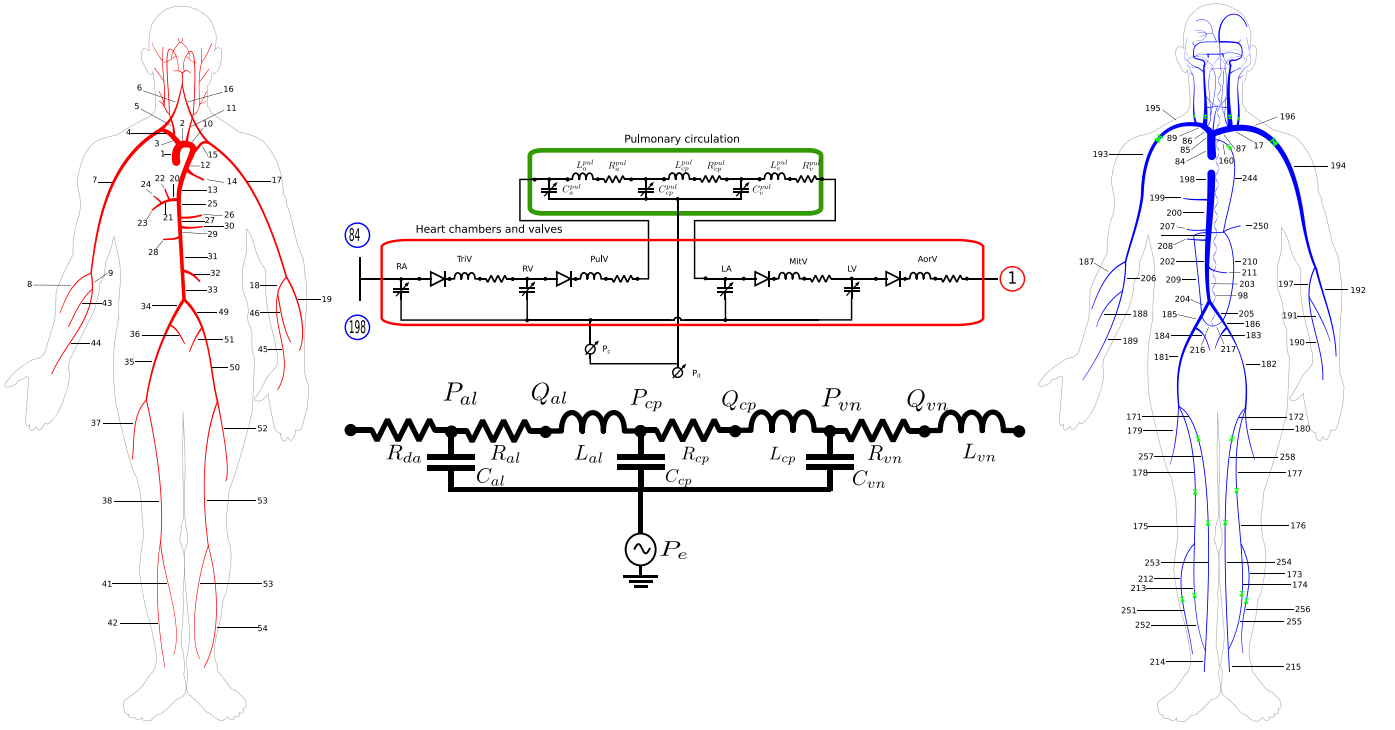

Fig. 1. Global, closed-loop model in Müller and Toro (2014). Major arteries and veins are modelled one-dimensionally; lumped-parameter models are used for heart, pulmonary circulation, arterioles, capillaries and venules. Left frame: major arteries; middle frame: lumped-parameter models; and right frame: major veins.