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Validated biomechanical model for efficiency and speed of rowing ☆

Peter F. Pelz *, Angela Vergé

Technische Universität Darmstadt, Germany, Chair of Fluid Systems, Magdalenenstr. 4, D-64289 Darmstadt, Germany

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ABSTRACT

The speed of a competitive rowing crew depends on the number of crew members, their body mass, sex and the type of rowing—sweep rowing or sculling. The time-averaged speed is proportional to the rower's body mass to the 1/36th power, to the number of crew members to the1/9th power and to the physiological efficiency (accounted for by the rower's sex) to the 1/3rd power. The quality of the rowing shell and propulsion system is captured by one dimensionless parameter that takes the mechanical efficiency, the shape and drag coefficient of the shell and the Froude propulsion efficiency into account. We derive the biomechanical equation for the speed of rowing by two independent methods and further validate it by successfully predicting race times. We derive the theoretical upper limit of the Froude propulsion efficiency for low viscous flows. This upper limit is shown to be a function solely of the velocity ratio of blade to boat speed (i.e., it is completely independent of the blade shape), a result that may also be of interest for other repetitive propulsion systems.

1. Introduction

The speed of a competitive rowing crew depends on the number of crew members, their body mass, sex and the type of rowing—sweep rowing or sculling. Since there are boats with 1, 2, 4, or 8 crew members there are in theory 32 rowing classes as shown schematically in Fig. 1.

Our objective is to derive and validate a general biomechanical equation for the time-averaged speed of rowing by taking into account the metabolic rate of the rower and all relevant loss mechanisms. The equation accounts for the number of crew members n (i.e., the number of prime movers), the body mass of the competitive rower m, the propulsion type (either sweep rowing or sculling), and the gender of the rowing crew. The literature survey is presented according to the four possible approaches that are found there: (1) empirical research; (2) detailed physical modeling; (3) time-averaged energy balance in integral form; and (4) dimensional analysis.

We use the independent approaches (3) and (4) to derive the same equation for the speed of rowing, whereas most research on the biomechanics of rowing has followed the first two approaches (Affeld, et. al., 1993; Kleshnev, 1999; Cabrera, et. al., 2006). There is no research on the influence of body mass on the boat speed. A dimensional analysis (McMahon, 1971) that takes the geometric similarity of the shells for different rowing classes into account yields the 1/9th power law for the relation between speed and number of rowers. For a dimensional analysis, it is typical only to consider continuous scales and magnitudes. In contrast, we apply the methods of dimensions, as Rayleigh described it (Rayleigh, 1877), to discrete variables, such as the number n of crew members. Here, we define a discrete measure N and require scale invariance, i.e. Bridgman's postulate (Bridgman, 1922), also for the number of crew members (Barenblatt, 2003). Although rowing is a biomechanical system, the prime mover (i.e. the oarsmen and, in particular, his or her weight), has been thus far left out of the equation. It is one of our main objectives to determine the dependency of the body mass of the competitive rowers on the average boat speed. The influence of the mass is twofold. On the one hand, due to allometric scaling, the input power increases with body mass to the power of 3/4th according to Kleiber's law (Kleiber, 1932 and 1975) and, in fact, we validate Kleiber's law for heavyweight and lightweight, male and female crews. On the other hand, as body mass increases, the shell surface increases due to Archimedes' law. This results in an increase in frictional drag. As will be seen, for a competitive rower, boat speed increases with body mass only to the power of 1/36th.

The outline of the paper is as follows. We first derive the equation for the speed of rowing and show the scale invariance of rowing. The identical result is then achieved independently by means of an energy balance for rowing that is analogous to Lighthill's analysis of fish movement (Lighthill, 1960). In the Section 3 "Validation of Allometric Scaling of Rowing", we confirm Kleiber's law by analyzing race times of winning crews at world

☆ *Dedicated to Professor Dieter Hellmann – a former oarsman – on the occasion of his 70th birthday.*

* Corresponding author. Tel.: +49 6151 162153; fax: +49 6151 162453
E-mail address: peter.pelz@fst.tu-darmstadt.de (P.F. Pelz).

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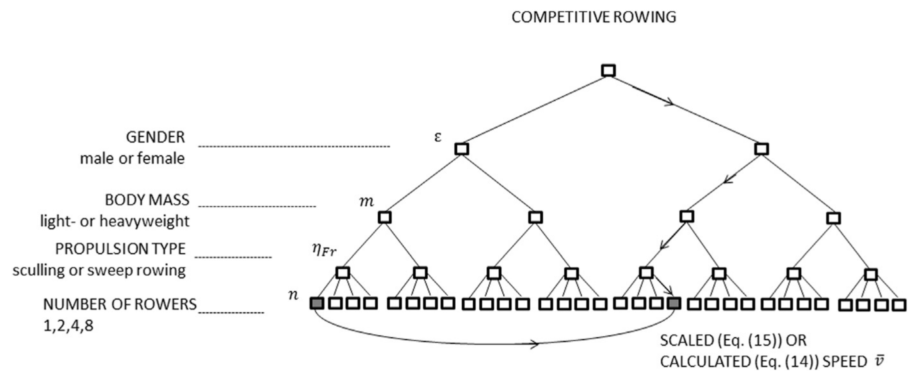

Fig. 1. There are in theory 32 possible rowing classes (neglecting cox) which differ in gender, body mass, propulsion type, number of crew members resulting in different average boat speed.

cup and Olympic regattas. In the final section 4 we use our results to predict the boat speed of various rowing classes at major regattas and validate the predictions.

2. Scale invariance and speed of rowing

There are three types of similarity and scaling: (1) geometric similarity/scaling; (2) physical similarity/scaling; and (3) allometric similarity/scaling. Geometric similarity is a special form of physical similarity based on Bridgman's postulate (Bridgman, 1922). Kleiber's law (Kleiber, 1932 and 1975), i.e. the metabolic rate and hence the mechanical power of an organism is proportional to its body mass raised to the power of 3/4$^{\text{th}}$ is an allometric scaling. This empirical relationship has been found to hold across the living world from bacteria to blue whales. Today, it is agreed that the 3/4$^{\text{th}}$ power can be explained by the specific geometry (West, et. al., 1999) or type (Banavar, et. al., 1999) of the metabolic networks, such as the geometry and topology of blood vessels. As far as we know there is no research on allometric scaling in rowing. As will be seen, combining all three scaling and similarity methods will lead us to the desired speed of rowing.

McMahon, (1971) was the first to notice the geometric similarity in rowing. For a shell, the most relevant geometric data are boat length L, surface area $A:=\kappa_A^2 L^2$, shell width $B:=\kappa_B L$, and displaced water volume $V:=\kappa_V^3 L^3$. The shape factors $\kappa_A, \kappa_B, \kappa_V$ are given in Table 1 for 23 different rowing classes. The shape factors show a maximum relative standard deviation of 3.7%. The first requirement of any shell is to support the weight of the crew. This is described by a force balance of the buoyancy (proportional to the water density ϱ) and the total weight of crew, cox, and shell according to Archimedes' principle. The ratio of the crew's total weight without cox to the overall weight including cox and boat, defined by $\kappa_M:=nm/\varrho V$ is 19%, with a relative standard deviation of 20%. This large variance is of only minor importance, since the dimensionless, volume-specific surface of the shell, defined by

$$\kappa:=\frac{A}{(V\kappa_M)^{2/3}}=\frac{\kappa_A^2}{\kappa_V^2\kappa_M^{2/3}}, \tag{1}$$

fully accounts for the shell's drag. It is equal to 12.09, with a relative standard deviation of only 4.9%. Since κ is nearly constant, the surface area scales to the power 2/3 with the number of oarsmen and body mass:

$$A=\kappa(V\kappa_M)^{2/3}=\kappa(nm/\varrho)^{2/3}. \tag{2}$$

Along with the *geometric similarity*, there is a *physical similarity* for the shell's drag for all rowing classes: The time-averaged drag force of the shell $\overline{D}$ (time average is indicated by an over bar) depends on the speed of rowing $\overline{v}$ relative to calm water, the density ϱ and kinematic viscosity ν of water, the specific gravity constant g, the stroke rate $1/\tau$, and the shape of the shell, which is the same for all rowing classes due to the discussed geometric similarity: $\overline{D}=\overline{D}(\varrho,\overline{v},A,\nu,g,\tau,shape)$. This equation remains the same, regardless of which fundamental units (Rayleigh, 1877; Bridgman, 1922) are used to express the 7 quantities. Since this is a dynamic problem, the fundamental dimensions of length, mass, and time [LMT], or their equivalents length, force, and time [LFT], are used preferentially. Due to the required scale invariance, the relation can be expressed equivalently using only 4 dimensionless parameters:

$$\overline{c}_D=\overline{c}_D(\overline{Re},\ \overline{Fr},\ \lambda, shape), \tag{3}$$

where $\overline{c}_D:=2\overline{D}/(\varrho\overline{v}^2 A)$ is the drag coefficient, $\overline{Re}:=\overline{v}L/\nu$ is the Reynolds number, $\overline{Fr}:=\overline{v}/\sqrt{gL}$ is the Froude number, and $\lambda:=\overline{v}/u=\overline{v}/\Omega l$ ($\Omega=2\pi/\tau$, outboard length l) is the dimensionless boat speed known as advance ratio (Newmann, 1977). The drag coefficient is nearly constant for all rowing classes $(2.65\pm.15)\times 10^{-3}$ (Mang, 2008). Thus, competitive rowing hulls exhibit not only *geometrical similarity,* but also an approximate *physical similarity*.

For the moment, we assume (to be validated in Section 3) that the mechanical power $\overline{P}_0$ of competitive rowers—whether lightweight or heavyweight, male or female—scales with their body masses m according to Kleiber's allometric scaling law (Kleiber, 1932 and 1975)

$$\overline{P}_0=\varepsilon\, m^{3/4}\,. \tag{4}$$

Since ε is a measure of the physiological quality of the rower, it is justified to name it *physiological efficiency*. Since $\overline{P}_0$ and m are rower-specific physical quantities, we introduce a *dimension N* to account for the number of oarsmen n. The *scale* can either be one rower or two rowers, counting in *units* of one, two, or more. Hence, together with the dimensions for dynamic systems length L, mass M, and time T, the suitable fundamental system of dimensions is [$LMTN$]. With the above-discussed physical similarity accounting for the shell's drag, the speed of rowing can be reduced to a function of the following dimensional quantities: The physiological efficiency ε and body mass m of the rower, the water density ϱ, and the number of crew members n:

$$\overline{v}=\overline{v}(\varepsilon,\ m,\ \varrho,\ n). \tag{5}$$

This equation must remain unchanged, regardless of the fundamental units used to express the five quantities. Bridgman's postulate, i.e. “the absolute meaning of relative quantities” (Bridgman, 1922), results in the scale invariance of Eq. (5). Hence, $\overline{v}=\overline{v}(\varepsilon,\ m,\ \varrho,\ n)$ is equivalent to a single dimensionless product $\Pi=const$. The dimension of speed is given by $[\overline{v}]=L^1T^{-1}$, that of density, by $[\varrho]=M^1L^{-3}$. Since mass and physiological efficiency