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Membrane tension of red blood cells pairwisely interacting in simple shear flow

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ABSTRACT

Flow-induced membrane tension contributes to the release of molecules by red blood cells (RBCs), and extremely high tension may cause haemolysis. Here, we investigated the membrane tension of RBCs during pairwise interactions in simple shear flow, given that pairwise interactions form the basis of many-body interactions. RBCs were modelled as capsules with a two-dimensional hyperelastic membrane, and large deformations were solved by the finite element method. Due to the small size of the RBCs, surrounding fluid motion was estimated as a Stokes flow and solved by the boundary element method. The results showed that the maximum isotropic tension appeared around the dimple of the biconcave surface and not around the rim. A comparison of the results with solitary cases indicated that the maximum principal tension and isotropic tension were significantly increased by cell–cell interaction effects. As the volume fraction of RBCs is large under physiological conditions, as well as in blood flow in vitro, cell–cell interactions must be analysed carefully when considering mechanotransduction and haemolysis in blood flow.

1. Introduction

The red blood cell (RBC) is one of the major components of blood, and its primary function is to transport oxygen and carbon dioxide by flowing through blood vessels. RBCs are easily deformed by fluid flow and can pass through the microcirculation due to their large deformability (Pries et al., 1992). Flow-induced RBC deformation affects the ability of the cell to release various molecules and regulate their concentrations in blood (Goldsmith et al., 1995). In particular, when RBCs are strongly deformed by the surrounding fluid flow, they release ATP (Wan et al., 2008). Flow-induced deformation may also lead to haemolysis. There is a causal relationship between the stress level in the membrane and haemolysis (Levertt et al., 1972). Thus, it is crucial to clarify the stresses in the membrane to gain a better understanding of the physiology and pathology of microcirculation.

Since the RBC membrane is very thin, only deformation of its median surface is considered. Accordingly, the stresses are integrated across the wall thickness and replaced by tensions. Although there have been a number of studies investigating the motion of RBCs in fluid flow both analytically (Skotheim and Secomb, 2007; Noguchi, 2009) and numerically (Freund and Orescanin, 2011; Le and Chiam, 2011; Ramanujan and Pozrikidis, 1998; Sui et al., 2008; Zhao and Shaqfeh, 2011), there have been few studies regarding flow-induced membrane tension. In Peng et al. (2011) and Pozrikidis (2005), the RBC membrane tensions flowing through a capillary was investigated. It was found that the membrane principal tension exhibited a plateau value around the front and rear end of the parachute shape of RBCs. In the case of shear flow, Pozrikidis (2003) has investigated RBC membrane tension with small deformation limits, i.e., the tumbling mode of the RBC. Thus, elastic tension during large-scale deformation in shear flow remains unclear despite its relevance to understanding the physiology and pathology of RBCs.

In our previous study (Omori et al., under review(a)), we investigated the membrane tensions of a single RBC in simple shear flow and discussed the effects of the applied shear rate and viscosity ratio of the internal to external fluids. In this study, we investigated the effects of cell–cell interactions on RBC membrane tension, given that the volume fraction of RBCs under physiological conditions is large and the cell–cell interactions likely affect the deformation and tension of RBCs. To clearly observe the cell–cell interaction effect, we investigated pairwise interactions of RBCs under conditions of simple shear flow. The hydrodynamic interaction of deformable capsules, cells and vesicles have been investigated (Jadhav et al., 2007; Lac et al., 2007; Levant et al., 2012; Tan et al., 2012). Lac et al. (2007) investigated the membrane tension of spherical capsules during pairwise interaction in shear flow, and showed that high principal tension appeared at the tips in the separating regime when the applied

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shear rate was small, whereas it appeared in a plane perpendicular to the direction of vorticity under conditions of high shear rate. In Omori et al. (under review(b)), we numerically investigated the semi-dilute RBC suspension rheology using stresslet during the cell–cell interactions of two RBCs. In the present paper, we applied the above method to investigate the membrane tension during the pairwise interaction in shear flow.

In Section 2, we describe the governing equations and numerical schemes. In Section 3, we first discuss the distributions of the principal tension and isotropic tension during the interaction and examine the effects of the applied shear rate and viscosity ratio of the internal and external liquids. We then present a comparison of the results with those of the solitary case. Section 4 concludes this study.

2. Problem statement

2.1. Governing equations

In this study, RBCs are modelled as capsules with a two-dimensional (2D) hyperelastic membrane. Since detailed capsule mechanics were presented previously by Barthés-Biesel et al. (2010) and Pozrikidis (1992), they are only derived in a brief outline here.

Assume that two RBCs are filled with an incompressible Newtonian liquid with density ρ and viscosity $\lambda\mu$ and freely suspended in another incompressible Newtonian liquid with the same density ρ, but viscosity μ. Due to the small size of an RBC, the inertial effects of the internal and external fluid flow can be neglected. The interfacial velocity $\boldsymbol{v}$ is given by the integral equation over the deformed membrane (Foessel et al., 2011; Pozrikidis, 1992)

$$\frac{1+\lambda}{2}\boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{v}^{\infty}(\boldsymbol{x}) - \frac{1}{8\pi\mu}\sum_{i=1}^{N_c}\int_{S_i}\boldsymbol{J}(\boldsymbol{x},\boldsymbol{y})\cdot\boldsymbol{q}(\boldsymbol{y})\,dS_i(\boldsymbol{y}) + \frac{1-\lambda}{8\pi}\sum_{i=1}^{N_c}\int_{S_i}^{PV}\boldsymbol{v}(\boldsymbol{y})\cdot\boldsymbol{K}(\boldsymbol{x},\boldsymbol{y})\cdot\boldsymbol{n}(\boldsymbol{y})\,dS_i(\boldsymbol{y}), \qquad (1)$$

where $\boldsymbol{v}^{\infty}$ is the undisturbed background flow, $\boldsymbol{n}$ is the outward unit normal vector, λ is the viscosity ratio of the internal to external liquid, N_c is the number of RBCs, and $\boldsymbol{J}$ and $\boldsymbol{K}$ are the Green functions of the single- and double-layer potentials, respectively. *PV* denotes the principal value of the double-layer integral (Pozrikidis, 1992).

Since the RBC membrane is very thin as compared with cell size, the membrane can be modelled as a 2D sheet without bending resistance. In this case, the stress is replaced by the tension (force per arch unit length) and the surface load $\boldsymbol{q}$ is given by the static equilibrium equation (Barthés-Biesel et al., 2010)

$$\boldsymbol{q} = -\nabla_s\cdot\boldsymbol{\tau}, \qquad (2)$$

where ∇_s is the surface gradient operator and $\boldsymbol{\tau}$ is the membrane tension. To model large deformations of the RBC, Skalak et al. (1973) proposed the law (SK), which is now commonly used for modelling the RBC membrane. The elastic principal tensions of the SK law τ_1 and τ_2 can be written as

$$\tau_1 = \frac{G_s\lambda_1}{\lambda_2}(\lambda_1^2 - 1 + C\lambda_2^2(\lambda_1^2\lambda_2^2 - 1)) \quad (\text{likewise for } \tau_2), \qquad (3)$$

where λ_i is the principal stretch ratio of the 2D membrane, G_s is the shear modulus of the membrane, and C is a dimensionless coefficient that measures the area dilation resistance. We use $C = 10$ in this study, which is a sufficiently large value to express the area incompressible property of the membrane (Walter et al., 2011).

2.2. Numerical methods

To compute pairwise interactions between the RBCs, finite element and boundary element coupling methods are used (Foessel et al., 2011; Walter et al., 2010). As a detailed numerical methodology and its accuracy were previously reported (Omori et al., 2012), we only present an outline here.

We track the membrane material point $\boldsymbol{x}$ over time, and thus the local membrane deformation can be computed readily by comparison with the reference state. The membrane tension $\boldsymbol{\tau}$ is obtained from Eq. (3). The finite element method (Walter et al., 2010) is employed to solve (2) with respect to $\boldsymbol{q}$. Once the load $\boldsymbol{q}$ is given, Eq. (1) can be solved by the boundary element method (Foessel et al., 2011). The membrane material point $\boldsymbol{x}$ is updated by the kinematic condition $d\boldsymbol{x}/dt = \boldsymbol{v}$. An explicit second-order Runge–Kutta method is used for the position update. The whole process is repeated while sufficiently long computational time.

Briefly, the membrane is discretised by 5120 linear triangles. A Cartesian frame (x,y,z) is set and the background shear flow is given by $\boldsymbol{v}^{\infty} = (\dot{\gamma}y,0,0)$, where $\dot{\gamma}$ is the shear rate.

To discuss the initial orientation of the RBC efficiently, we define two angles θ^0 and ϕ^0 [cf. Fig. 1(a)]. The angle θ^0 is the initial angle between the revolution axis of the RBC and the vorticity direction. Thus, the revolution axis corresponds to the z-axis when $\theta^0 = 0$, while it corresponds to the y-axis when $\theta^0 = \pi/2$. Since the membrane properties become symmetric in the flow and vorticity direction when $\theta^0 = 0$, the RBC showed a steady tank-treading motion. When $\theta^0 = \pi/2$, on the other hand, the RBC showed a swinging motion because of the non-isotropic properties of the membrane in the flow direction. The angle ϕ^0 is also defined as the initial angle between the major axis and the flow direction to control the contact phase of the RBCs under swinging motion.

The relative position of the two RBCs $\Delta\boldsymbol{x}$ is defined as $\Delta\boldsymbol{x} = \boldsymbol{G}_1 - \boldsymbol{G}_2$, where $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ are the centres of mass of RBC1 and RBC2, respectively [cf. Fig. 1(b)], and are calculated from the

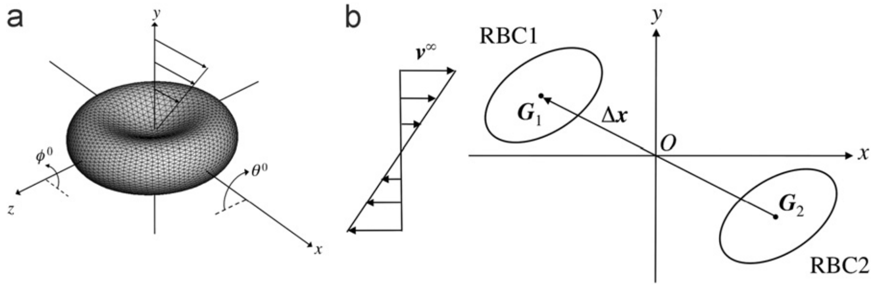

Fig. 1. (a) The initial inclination angle ϕ^0 is defined as the angle between the flow direction and the major axis of the RBC. The angle θ^0 is defined as the initial angle between the z-axis and the revolution axis of the RBC. (b) Schematic illustration of the cell–cell interaction between the two RBCs.