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Predicting human walking gaits with a simple planar model

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A R T I C L E I N F O

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Models of human walking with moderate complexity have the potential to accurately capture both joint kinematics and whole body energetics, thereby offering more simultaneous information than very simple models and less computational cost than very complex models. This work examines four- and six-link planar biped models with knees and rigid circular feet. The two differ in that the six-link model includes ankle joints. Stable periodic walking gaits are generated for both models using a hybrid zero dynamics-based control approach. To establish a baseline of how well the models can approximate normal human walking, gaits were optimized to match experimental human walking data, ranging in speed from very slow to very fast. The six-link model well matched the experimental step length, speed, and mean absolute power, while the four-link model did not, indicating that ankle work is a critical element in human walking models of this type. Beyond simply matching human data, the six-link model can be used in an optimization framework to predict normal human walking using a torque-squared objective function. The model well predicted experimental step length, joint motions, and mean absolute power over the full range of speeds.

1. Introduction

At one extreme of modeling human walking are very basic models (Kuo, 2007) that typically approximate one aspect of gait using a simple mechanical system. Examples include modeling the swing leg as a pendulum (Doke et al., 2005) and modeling the step-to-step transition with a two-link, point-foot biped (Donelan et al., 2002). Such models clearly demonstrate the key underlying gait mechanics, and their straightforward mathematics lead to easily verifiable predictions that, when tested with human subject experiments, are consistently accurate (Kuo, 2007).

At the other end of the spectrum are detailed forward dynamics models (Anderson and Pandy, 2001; Liu et al., 2008). When driven by muscles, such models can help determine how muscles affect the energy flow among the limbs (Zajac et al., 2002) and how muscle forces change with differing conditions (Neptune et al., 2008), ultimately providing insight into muscle synergies and coordination. Model control either determines muscle forces to reproduce motion capture data (Thelen and Anderson, 2006) or assumes an objective function for forward simulation (Ackermann and van den Bogert, 2010). The complicated nature of these systems limits the number of cases examined because both the optimization and simulation are time consuming.

Models between the two extremes can be complex enough to capture multiple gait features simultaneously, yet simple enough to enable rapid motion optimization. Planar models controlled based on hybrid zero dynamics (HZD, Westervelt et al., 2007) are one example. HZD-based control parameterizes the desired joint trajectories as functions of a phase variable that measures step progression, and not of time explicitly. Prior work has shown that an HZD-inspired model can accurately match human hip and knee kinematics at one walking speed (Srinivasan et al., 2008) and translate human gait to robotic gait (Sinnet and Ames, 2012).

To date, predictive simulations of normal human walking that capture more than one aspect of gait have typically been used only to model the self-selected walking speed with step length constrained (Anderson and Pandy, 2001; Ren et al., 2007; Xiang et al., 2011). Simulated joint kinematics generally show reasonable agreement with human data, although quantitative results are rarely provided. Because speed-related gait changes are well documented (Kirtley et al., 1985; Lythgo et al., 2011; Oberg et al., 1994), the ability to predict gait at the self-selected speed does not necessarily translate into accurate gait predictions across a wide range of speeds. Such predictions are necessary for the design and selection of mechanical interventions, such as prostheses, which motivates the predictive model development and validation herein.

The model is grounded in HZD-based control because it offers an appealing compromise between complexity and simulation speed and prior work has suggested that humans control their motion based on phase variables (Grasso et al., 1998; Gregg et al., 2014). However, HZD-based control was originally developed for point-foot

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robots (such as Yang et al., 2008), so a means of accounting for the human ankle-foot complex is needed. Due to deformation, modeling the stance foot as a flat plate jointed to the shank is unlikely to yield accurate results with an HZD-based model. Rather, a curved foot is favored since the center of pressure (CoP) trace forms a circular arc through most of stance when plotted in a shank-fixed coordinate frame (Hansen et al., 2004). Srinivasan et al. (2008) take this approach, with a curved foot rigidly attached to the stance shank. Without an ankle joint, however, the contributions of the ankle-foot complex in controlling whole body center of mass (CoM) acceleration (Anderson and Pandy, 2003) and in redirecting the CoM velocity in an energetically optimal manner during the step-to-step transition (Donelan et al., 2002) are lost.

This work defines HZD-based curved foot models with and without ankles, demonstrates that the model with ankles is more accurate in modeling normal human walking, establishes that this model can mimic normal human walking at speeds ranging from very slow to very fast, and identifies an objective function that can be used to accurately predict normal human walking with it.

2. Methods

2.1. Models

The planar four-link model consists of revolute hip and knee joints connecting two thighs and two shanks, both with mass and inertia, and a point mass at the hip representing the mass of the head, arms and trunk. Constant radius circular feet are rigidly attached to the shanks with their centers of curvature located anterior to the shanks (Hansen et al., 2004). The six-link model is identical except the feet, having mass but no inertia, are pinned to the shanks with revolute joints (Fig. 1). Representing the degrees of freedom in a model, $N=4$ for the four-link model (6 for the six-link). The primary purpose of including ankle joints is to allow positive work at the stance ankle that represents the effects of both ankle plantar flexion and toe extension in humans. In addition to capturing toe extension, the model ankle differs from the human ankle in that the circular foot shape itself achieves some of the human ankle's functionality by capturing much of the CoP movement relative to the shank. As a result, the model ankle motion is expected to differ somewhat from the physiological ankle motion. Ideal torque generators actuate the leg joints (q_2-q_N), with a single hip actuator controlling the angle between the thighs (q_2 in Fig. 1). The point of contact between the foot and ground is unactuated, and the unactuated motion is captured in the stance hip absolute angle (q_1 in Fig. 1).

The HZD-based control approach for curved foot bipeds was applied to the model (Martin et al., 2014). Gaits are one-step periodic, with each step consisting of a finite-time single support phase and an instantaneous double support phase. While humans have a finite-time double support phase, condensing it to an instant simplifies the control problem significantly without compromising the model's ability to capture the dominant gait characteristics. Double support is modeled as an impulsive transition during which the pre-impact swing/stance leg contacts/lifts off the ground. This impact results in an instantaneous change in joint velocities and a swap of leg roles,

$$\mathbf{q}^+ = \mathbf{S}\mathbf{q}^- \tag{1}$$

$$\dot{\mathbf{q}}^+ = \mathbf{A}(\mathbf{q}^-)\dot{\mathbf{q}}^-, \tag{2}$$

where $\mathbf{q}$ is the $N\times 1$ joint angle vector, $\mathbf{S}$ is an $N\times N$ matrix that switches the angle definitions and $\mathbf{A}$ is an $N\times N$ matrix relating the pre- and post-impact velocities. The superscripts '−' and '+' refer to the instants immediately before and after impact, respectively.

During single support, the dynamic equations can be integrated forward in time:

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{u}, \tag{3}$$

where $\mathbf{D}$ is the $N\times N$ inertia matrix, $\mathbf{C}$ is the $N\times N$ matrix containing Coriolis and centripetal terms, $\mathbf{G}$ is the $N\times 1$ vector containing gravity terms, and $\mathbf{u}$ is the $N\times 1$ vector of joint torques. Using a nonlinear, gait-specific coordinate transformation and appropriate joint torques, Eq. (3) can be changed such that it is composed of a linear system involving the joint torques and a nonlinear, passive system (Westervelt et al., 2007). If this passive system is stable and periodic, which can be evaluated using closed form equations, the whole system is stable and periodic. The joint torques $\mathbf{u}$ are chosen to achieve the desired motion $\mathbf{y}$ of the actuated joints. It is critical that $\mathbf{y}$ be a function of only the joint angles (not of time) and the unactuated angle. A convenient method is to define $\mathbf{y}$ using fifth order Bézier polynomials:

$$y_j = q_j - \sum_{k=0}^{5} \alpha_{kj}\frac{5!}{(5-k)!k!}s^k(1-s)^{5-k}, \tag{4}$$

where $j=1,2,\ldots N-1$, $\alpha_{k,j}$ denotes the control parameters to be chosen and $0\le s\le 1$ is the phase variable, in this case a linearized approximation of the horizontal hip position, normalized so that $s=0$ and $s=1$ correspond to the start (impact) and end of the step (lift-off), respectively. Since s is a function of the joint angles and the unactuated angle, the evolution of the desired joint angles is driven by hip progression, not by time explicitly. MATLAB code for finding and simulating periodic gaits is available in multimedia extension one.

2.2. Matching human walking

Published experimental walking data from several sources (Bianchi et al., 1998; Borghese et al., 1996; Bovi et al., 2011; Han and Wang, 2011; Murray et al., 1984) over a wide range of speeds were used to design gaits to evaluate how well each model could match normal human walking ($n=16$). Only papers reporting overground walking for healthy subjects at a minimum of two speeds without enforcing a specified step cadence or step length (to avoid unnatural gait alternations, Bertram, 2005) were included. Various sources used different optical-based protocols to measure joint angles, but the differences in reported joint angles are small (Ferrari et al., 2008). Both child and adult data were included because gait is known to be mature by age 7 (Sutherland, 1997; Ganley and Powers, 2005; Hansen and Meier, 2010). Because of the range of subject sizes, all reported data were nondimensionalized using the factors proposed in Hof (1996). For reference, the nondimensional self-selected speed is about 0.42, and the nondimensional walk-to-run transition speed is about 0.71.

Plots of joint kinematics were digitized using Engauge Digitizer (Mitchell, 2002), and piecewise cubic Hermite interpolation was applied to obtain 100 evenly spaced data points. In a worst-case test, errors were well under 1°, indicating sufficient digitization accuracy considering that the pointwise standard deviation of the joint angles is about 5° (Bovi et al., 2011; Schwartz et al., 2008; Winter, 1984). Because the models have an instantaneous double support phase, each human double support phase was divided evenly between the model stance and swing phases. Only nondimensional speeds greater than 0.2 were included because the double support phase is too long at slower speeds for the model to accurately capture the gait dynamics.

Subjects had differing body mass indexes, so subject-specific models were created using the given height and mass, standard anthropometric tables (Winter, 2009) and the curved foot properties from Hansen et al. (2004). Simulations were conducted using a fully dimensioned model, and results were nondimensionalized as needed. For each experimental gait, an optimization was used to design a model gait that minimized the error in matching the experimental kinematics by varying the control parameters $\alpha_{j,k}$ from Eq. (4). (Because of differences between the model and the human, simply matching the reported joint kinematics does not yield periodic steps.) No explicit effort was made to match human kinetics. The objective function was

$$f = 10^4\underbrace{(S_{sim}-S_{exp})^2}_{\text{Step length}} + 10^4\underbrace{(v_{sim}-v_{exp})^2}_{\text{Speed}}\ldots$$

$$+\underbrace{\|\alpha_{sim,\ \text{Hip and Knee}} - \alpha_{exp,\ \text{Hip and Knee}}\|_2}_{\text{Actuated angles}}\cdots$$

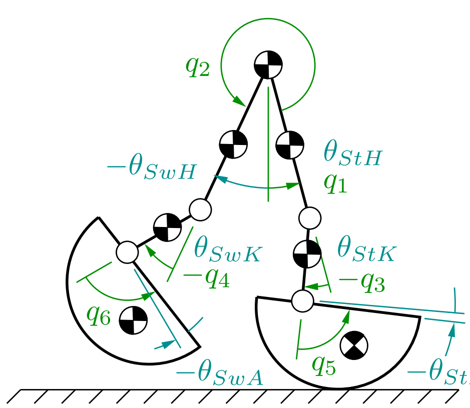

Fig. 1. Schematic of the six-link model. For the four-link model, the feet are rigidly attached perpendicular to the shanks rather than jointed to them with ankles, as in the six-link model. The joint angles used in the model are indicated with the q_i's. The unactuated angle is q_1, and the actuated angles are q_2 through q_6. The plots, however, transform the model angles into a more typical biomechanics convention as indicated with the θ_i's.