Journal of Biomechanics 47 (2014) 1742–1747

Contents lists available at ScienceDirect

Journal of Biomechanics

journal homepage: www.elsevier.com/locate/jbiomech
www.JBiomech.com

Short communication

Point of optimal kinematic error: Improvement of the instantaneous helical pivot method for locating centers of rotation

Helios De Rosario [a,c,*], Álvaro Page [b,c], Vicente Mata [d]

[a] Instituto de Biomecánica de Valencia, Universitat Politècnica de València, Edificio 9C, Camino de Vera s/n, E-46022 Valencia, Spain
[b] Departamento de Física Aplicada, Universitat Politècnica de València, Spain
[c] Grupo de Tecnología Sanitaria del IBV, CIBER de Bioingeniería, Biomateriales y Nanomedicina (CIBER-BBN), Valencia, Spain
[d] Departamento de Ingeniería Mecánica y de Materiales, Universitat Politècnica de València, Spain

ARTICLE INFO

Article history:
Accepted 5 February 2014

Keywords:
Center of rotation
Instantaneous helical axis
Functional calibration
Optimization

ABSTRACT

This paper proposes a variation of the instantaneous helical pivot technique for locating centers of rotation. The point of optimal kinematic error (POKE), which minimizes the velocity at the center of rotation, may be obtained by just adding a weighting factor equal to the square of angular velocity in Woltring's equation of the pivot of instantaneous helical axes (PIHA). Calculations are simplified with respect to the original method, since it is not necessary to make explicit calculations of the helical axis, and the effect of accidental errors is reduced. The improved performance of this method was validated by simulations based on a functional calibration task for the gleno-humeral joint center. Noisy data caused a systematic dislocation of the calculated center of rotation towards the center of the arm marker cluster. This error in PIHA could even exceed the effect of soft tissue artifacts associated to small and medium deformations, but it was successfully reduced by the POKE estimation.

1. Introduction

A well known method for locating the center of rotation (CoR) of a ball joint consists of calculating the "pivot" point of the instantaneous helical axes (IHA) of a set of calibration gestures. This method was first proposed by Woltring (1990), and it is still very much used, specially for the gleno-humeral joint after the recommendation of the International Society of Biomechanics (Wu et al., 2005). The conventional procedure consists of three steps. First, calculate the instantaneous kinematic parameters of the relative motion between the linked segments, defined by the angular velocity ($\boldsymbol{\omega}_t$) and the velocity of an arbitrary point $\mathbf{p}_t$ ($\dot{\mathbf{p}}_t$), for each instant $t=1, \ldots, n$. Second, calculate IHA positions as:

$$\mathbf{s}_t = \dot{\mathbf{p}}_t + \frac{\boldsymbol{\omega}_t \times \dot{\mathbf{p}}_t}{\omega_t^2} \tag{1}$$

And third, calculate the pivot of IHA (PIHA) by solving the following equation:

$$\left(\sum_{t=1}^{n} \mathbf{Q}_t\right)\mathbf{s}_{PIHA} = \sum_{t=1}^{n} \mathbf{Q}_t \mathbf{s}_t, \tag{2}$$

where $\mathbf{Q}_t$ are orthogonal projection matrices, defined by the unit vectors of $\boldsymbol{\omega}_t$ ($\mathbf{n}_t$) and the identity matrix $\mathbf{I}$:

$$\mathbf{Q}_t = \mathbf{I} - \mathbf{n}_t \mathbf{n}_t^T \tag{3}$$

This "averaging" procedure cancels out IHA errors that present opposing directions during the calibration movements. However, its precision is challenged by the high sensitivity of IHA errors to low angular velocities. This may be solved by discarding all the instants where ω_t is below a threshold, often set at 0.25 rad/s for measures taken between 10 and 50 Hz (Monnet et al., 2007; Stokdijk et al., 2000; Veeger and Yu, 1996).

An alternative proposed by Halvorsen et al. (1999) consists of calculating the pivot of the finite helical axis (FHA), which defines the locus of minimum displacement from a reference position (Woltring et al., 1985). This variant has become very used too, and its numerical properties have been studied in detail. FHA are very sensitive to small rotations (instead of small velocities), but this is normally solved by including a weighting factor w_t (do not confound with angular velocities) depending on the rotation angle ϑ_t. An optimal solution has been found in $w_t = \sin^2(\vartheta_t/2)$, which gives the minimum error in terms of CoR displacement (Ehrig et al., 2006).

In this paper we propose a similar optimization of the PIHA method, which manages more effectively the sensitivity of IHA errors to angular velocities. This hypothesis was validated by a simulation, modeled upon a real measurement of the gleno-humeral joint.

* Corresponding author at: Instituto de Biomecánica de Valencia, Universitat Politècnica de València, Edificio 9C, Camino de Vera s/n, E-46022 Valencia, Spain. Tel.: +34 963879160; fax: +34 963879169.
E-mail address: helios.derosario@ibv.upv.es (H. De Rosario).

http://dx.doi.org/10.1016/j.jbiomech.2014.02.003
0021-9290

2. Materials and methods

2.1. Optimization of the PIHA method

Like its variant based on FHA, Woltring's method may be optimized by adding a weighting factor equal to ω_t^2, which ensures the minimum error in terms of relative velocity in the CoR (see Supplementary Appendix A.1). Thus, (2) may be rewritten to give the point of optimal kinematic error (POKE):

$$\left(\sum_{t=1}^{n} \omega_t^2 \mathbf{Q}_t\right)\mathbf{s}_{POKE} = \sum_{t=1}^{n} \omega_t^2 \mathbf{Q}_t \mathbf{s}_t \tag{4}$$

This equation may be simplified, avoiding the explicit calculation of IHA, by setting the reference point $\mathbf{p}_t$ at the origin of coordinates for any t, and scaling $\mathbf{Q}_t$ by ω_t^2:

$$\mathbf{W}_t = \omega_t^2 \mathbf{Q}_t = \omega_t^2 \mathbf{I} - \boldsymbol{\omega}_t \boldsymbol{\omega}_t^T \tag{5}$$

Then, (4) becomes:

$$\left(\sum_{t=1}^{n} \mathbf{W}_t\right)\mathbf{s}_{POKE} = \sum_{t=1}^{n} \boldsymbol{\omega}_t \times \dot{\mathbf{p}}_t \tag{6}$$

2.2. Experimental validation

A subject signed an informed consent to participate in the experiment. He was instrumented with three markers placed on the right acromion to measure the scapular motion (Karduna et al., 2001), and three markers on the skin of the upper arm. Arm markers had its center at 150 mm from the acromion, and they were separated about 110 mm from each other, although those distances varied due to STA. The subject made five consecutive cycles of typical functional calibration gestures: arm flexion–extension, elevation, and half-circumduction, with a maximum elevation of 45° (Leardini et al., 1999).

These motions were recorded by 10 cameras at 50 Hz, with a photogrammetry system (Kinescan/IBV). The rigid rotations of the humerus w.r.t. the scapula and the deformation of the humeral marker cluster were extracted from these measures, as by De Rosario et al. (2012). The ranges of marker displacements within the bone frame are presented in Table 1. All measures were defined in local coordinates systems, that were aligned with the global reference frame (Wu and Cavanagh, 1995) when the subject adopted the reference posture (upright, arms at sides and palms facing forward).

A theoretical motion of the humeral markers was then simulated, repeating the measured rotation patterns, and assuming a joint center at (40, −40, −10) mm from the acromion, based on Stokdijk et al. (2000). That "ideal" center was used as reference point $\mathbf{p}_t$, so that the calculated centers would measure the CoR errors. The ideal motion was altered by a continuous noise based on Begon et al.'s (2007) model: marker positions were modeled as Gaussian functions of the motion cycle to simulate soft tissue artifacts (STA), disturbed by white noise (see Supplementary Appendix A.2). STA were defined from the values of Table 1, scaled by a factor equal to 0 (null artifact), 0.5 (small artifacts, with maximal marker displacements around 4.5 mm), or 1 (medium artifacts, with maximum displacements around 9.0 mm). The standard deviation of white noise (σ) ranged from 0 to 1 mm, in 0.1 mm steps.

Each combination of STA and noise sizes was simulated 100 times. Marker positions and velocities were calculated from noisy data by a local polynomial filter of 7th order. The filter's bandwidth was $N = 13$ samples for an optimal calculation

Table 1
Maximum and minimum marker displacements by deformation (in mm), for markers M1, M2, M3, in the three coordinates of the humerus frame. The maximum absolute values of each range are marked with an asterisk.

		M1			M2			M3		
		x	*y*	*z*	*x*	*y*	*z*	*x*	*y*	*z*
Fl.–ext.	Min	−1.1	−3.7	−3.5*	−0.5	−2.9	−2.8*	−1.4*	−1.7*	−3.5
	Max	1.6*	4.0*	2.4	1.0*	4.4*	2.2	1.2	1.3	4.8*
Elev.	Min	−0.5	−2.4*	−6.0*	−1.9*	−1.1	−1.4	−0.7*	−3.7*	−1.0
	Max	2.0*	1.5	1.0	0.5	4.9*	3.5*	0.3	0.5	3.0*
Half circ.	Min	−1.3	−5.6*	−6.4*	−1.4*	−3.4	−2.0	−1.8*	−2.8*	−3.6
	Max	2.8*	4.1	2.0	0.8	7.4*	3.1*	1.2	0.3	6.9*

Fl.–ext.: flexion extension; Elev.: elevation and Half circ.: half circumduction.

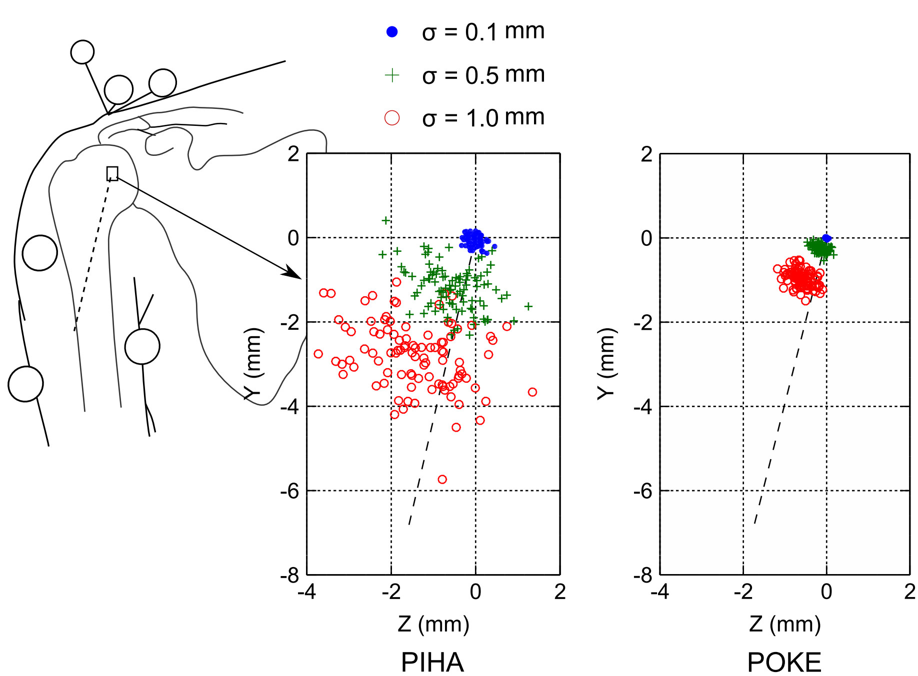

Fig. 1. CoR errors in the *YZ*-plane, for null STA and three different noise levels. The dashed black line represents the separation between the error-free CoR and the center of the marker set. The measures inside the plot frame (in mm) are exact; the sketch of the subject's shoulder and upper arm is an approximate representation to provide a visual clue of the proportions.